

**Section 8.5**  
**The Four Color Theorem**

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**FOUR COLOR CONJECTURE**

The four color conjecture dates back to 1852 when a student, Francis Guthrie, of Augustus de Morgan asked de Morgan to verify the “fact” that any map drawn in the plane could be colored with at most four colors, so that adjacent (that is, sharing a boundary) countries received different colors. De Morgan responded by saying he did not know that it was a “fact.”

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**THE FOUR COLOR CONJECTURE IN GRAPH-THEORETIC TERMS**

**The Four Color Conjecture:** Every planar graph can be colored with four or fewer colors.

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**HEAWOOD'S FIVE-COLORING  
THEOREM**

**Theorem 8.5.1 (Heawood):** Every planar graph is 5-colorable.

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**THE FOUR COLOR THEOREM**

**Theorem 8.5.2 (The Four Color Theorem):**  
Every planar graph is 4-colorable.

NOTES:

- The Four Color Theorem was not proven until 1976 by Kenneth Appel and Wolfgang Haken.
- The proof is very long and has many cases.
- It required almost 1200 hours of computer time to check that these cases worked.

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