Section 8.1

Vertex Independence and Coverings

RELATION BETWEEN INDEPENDENT VERTICES AND COVERINGS

Proposition 8.1.1: In a graph $G = (V, E)$, a subset I of V is independent if and only if $V - I$ is a covering of G.

MAXIMUM AND MAXIMAL INDEPENDENT SETS

- An independent set in G is called a **maximum** independent set provided no other independent set in G has larger cardinality.
- An independent set in G is called **maximal** if it is contained in no larger independent set.

INDEPENDENCE AND COVERING NUMBERS

- The number of vertices in a maximum independent set in G is called the **independence number** of G and is denoted by $\beta(G)$.
- The number of vertices in a minimum covering of G is called the **covering number** of G and is denoted by $\alpha(G)$.
- The **edge independence number**, denoted $\beta_1(G)$, is the size of a maximum matching in G.
- The **edge covering number**, denoted by $\alpha_1(G)$, is the minimum size of a set L of edges with the property that every vertex is an end vertex of some edge in L.

RELATIONSHIP BETWEEN INDEPENDENCE AND COVERING NUMBERS

Theorem 8.1.1 (Gallai's Theorem): If G is a graph of order p with $\delta(G) > 0$, then

$$\alpha(G) + \beta(G) = p \quad \text{and}$$

$$\alpha_1(G) + \beta_1(G) = p.$$

BIPARTITE GRAPHS

Theorem 8.1.2: If G is a bipartite graph with $\delta(G) > 0$, then $\beta(G) = \alpha_1(G)$.