Section 6.1

Euler's Formula

PLANAR GRAPHS

- A (p, q) graph G is said to be **embeddable in the plane** or **planar** if it is possible to draw G in the plane so that the edges of G intersect only at end vertices.
- If such a drawing has been done, we say that a **plane embedding** of the graph has been found.

REGIONS OF PLANAR GRAPHS

- Given a plane embedding of the graph G, a **region** of G is a maximal section of the plane for which two points may be joined by a curve.
- Intuitively, a region is a connected section of the plane bounded (often enclosed) by some set of edges of G.
- The region of the plane that is not enclosed by a set of edges of G is called the **exterior region**.

EULER'S FORMULA

Theorem 6.1.1 (Euler): If G is a connected plane (p, q) graph with r regions, then $p - q + r = 2$.

MAXIMAL PLANAR GRAPHS; TRIANGULATED PLANAR GRAPHS

- The graph G is a **maximal planar graph** if G is planar but $G + xy$ is not planar for every pair of nonadjacent vertices x and y in $V(G)$.
- Since we can always add an edge between to nonadjacent vertices if a region is bounded by four or more edges, maximal planar graphs are sometimes referred to as **triangulated planar graphs** or simply **triangulations**.

A THEOREM ON MAXIMAL PLANAR GRAPHS

Theorem 6.1.2: If G is a maximal planar (p, q) graph with $p \geq 3$, then

$$q = 3p - 6.$$

FOUR COROLLARIES

Corollary 6.1.1: If G is a planar (p, q) graph with $p \geq 3$, then

$$q \leq 3p - 6.$$

Corollary 6.1.2: Every planar graph G contains a vertex of degree at most 5; that is, $\delta(G) \leq 5$.

Corollary 6.1.3: The graph $K_{3,3}$ is nonplanar.

Corollary 6.1.4: The graph K_5 is nonplanar.