Section 5.4

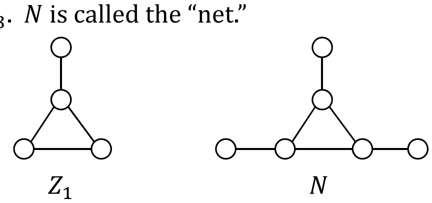
Forbidden Families

F-FREE GRAPHS

- If $F = \{H_1, H_2, \ldots, H_k\}$ is a family of graphs, we say that G is ***F*-free** if G does not contain any of the graphs in the set F as induced subgraphs.
- If F is a single graph H, we simply say G is H-free.

THE GRAPHS Z_1 and *N*

- We define $Z_1 = K_{1,3} + e$.
- We define the graph N to be C_3 with one edge and vertex attached to each vertex of C_3. N is called the "net."



A HAMILTONIAN RESULT AND FORBIDDEN FAMILIES

Theorem 5.4.1: If G is a 2-connected $\{K_{1,3}, Z_1\}$-free graph, then G is hamiltonian.

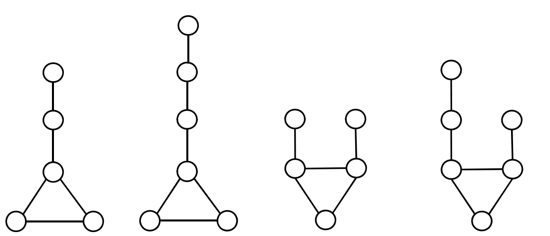
ANOTHER HAMILTONIAN RESULT AND FORBIDDEN FAMILIES

Theorem 5.4.2: If G is

1. connected and $\{K_{1,3}, N\}$-free, then G is traceable.
2. 2-connected and $\{K_{1,3}, N\}$-free, then G is hamiltonian.

Corollary 5.4.1: If G is a 2-connected graph that is $\{R, S\}$-free where $R = K_{1,3}$ and S is any one of $N, C_3, Z_1, B, P_4, P_3, K_2$ or K_1, then G is hamiltonian.

THE GRAPHS Z_2, Z_3, B and W



Z_2 Z_3 B "the bull" W "the wounded"

MORE HAMILTONIAN RESULT AND FORBIDDEN FAMILIES

Theorem 5.4.3: If G is a 2-connected graph and G is

1. $\{K_{1,3}, P_6\}$-free
2. $\{K_{1,3}, W\}$-free, or
3. $\{K_{1,3}, Z_3\}$-free and of order $p \geq 10$.

then G is hamiltonian.

Theorem 5.4.4: Let R and S be connected graphs ($R, S \neq P_3$) and G a 2-connected graph of order $n \geq 10$. Then G is $\{R, S\}$-free implies G is hamiltonian if and only if $R = K_{1,3}$ and S is one of the graphs $C_3, P_4, P_5, P_6, Z_1, Z_2, Z_3, B, N$, or W.