Section 5.3

Related Hamiltonian-like Properties

TRACEABLE AND HAMILTONIAN CONNECTED GRAPHS

- A graph is **traceable** if it contains a hamiltonian path.
- A graph G is called an **homogenously traceable** if there is a hamiltonian path beginning at every vertex of G.
- A graph G is called **hypohamiltonian** if G is not hamiltonian but $G - v$ is hamiltonian for every vertex v.
- We say that G is **hamiltonian connected** if every two vertices of G are joined by a hamiltonian path.

(p + 1)-CLOSURE

For a (p, q) graph G, let the **(p + 1)-closure**, denoted by $CL_{p+1}(G)$, be the graph obtained from G by recursively joining pairs of nonadjacent vertices whose degree sum is at least $p + 1$.

A THEOREM ON HAMILTONIAN CONNECTEDNESS

Theorem 5.3.1 (Bondy and Chvátal): Let G be a graph of order p. If $CL_{p+1}(G)$ is complete, then G is hamiltonian connected.

TWO COROLLARIES

Corollary 5.3.1: If G is a graph of order p such that for every pair of distinct nonadjacent vertices x and y in G, $deg\ x + deg\ y \geq p + 1$, then G is hamiltonian connected.

Corollary 5.3.2: If G is a graph of order p such that, $deg\ x \geq \frac{p+1}{2}$, then G is hamiltonian connected.

PANCONNECTED

A connected graph $G = (V, E)$ is said to be **panconnected** if for each pair of distinct vertices x and y, there exists and $x - y$ path of length l, for each l satisfying $d(x, y) \leq l \leq |V| - 1$.

Theorem 5.3.2: If G is a graph of order $p \geq 4$ such that for every vertex $x \in V(G)$, $deg\ x \geq \frac{p+2}{2}$, then G is panconnected.

PANCYCLIC

A graph G of order p is **pancyclic** if it contains a cycle of every length l, $(3 \leq l \leq p)$. G is **vertex pancyclic** if each vertex of G lies on a cycle of each length l, $(3 \leq l \leq p)$.

Theorem 5.3.3: If G is a hamiltonian (p, q) graph with $q \geq \frac{p^2}{4}$, then either G is pancyclic or p is even and G is isomorphic to $K_{p/2,\,p/2}$.