Section 4.1

Flows

NETWORKS

A **network** N is a 5-tuple $N = (V, E, s, t, c)$ where (V, E) is a digraph with distinguished vertices s and t called the **source** and **sink**, respectively. Each arc e is assigned a nonnegative real number $c(e)$, called the **capacity** of the arc; that is, c is a function which assigns nonnegative real numbers to the arcs of N.

Note: $in(v)$ and $out(v)$ denote the sets of arcs entering and leaving v, respectively.

FLOW

A (legal) **flow** f in N is a mapping from the arc set E to the real numbers such that:

1. (Capacity constraint) $f(e) \leq c(e)$ for every $e \in E$
2. (Flow constraint) for each vertex v other than s or t,

$$0 = \sum_{e \in in(v)} f(e) - \sum_{e \in out(v)} f(e)$$

LOOPS AND MULTIPLE ARCS

- Loops never add to the flow because what flows out of the vertex immediately flows back into the same vertex.
- Parallel arcs add nothing since we can replace parallel arcs with a single arc whose flow (and capacity) is the sum of flows (and capacities) on all parallel arcs.
- These restrictions yield $|E| \leq |V|(|V| - 1)$.

TOTAL FLOW

Given a network N with flow f, the **total flow** (sometimes called the value of the flow) is defined as:

$$F = \sum_{e \in in(t)} f(e) - \sum_{e \in out(t)} f(e)$$

That is, F is the net flow into the sink t. We want to maximize F.