Section 3.1

Fundamental Properties of Trees

TREES

Recall, that a **tree** is a connect acyclic graph.

TWO CHARACTERIZATIONS OF TREES

Theorem 3.1.1: A graph T is a tree if and only if every two distinct vertices of T are joined by a unique path.

Theorem 3.1.2: A (p, q) graph T is a tree if and only T is connected and $q = p - 1$.

CHARACTERIZATIONS OF TREES

Theorem 3.1.3: The following are equivalent on a (p, q) graph T:

1. The graph T is a tree.
2. The graph T is connected and $q = p - 1$.
3. Every pair of distinct vertices of T is joined by a unique path.
4. The graph T is acyclic and $q = p - 1$.

VERTICES AND EDGES IN TREES

- In any tree of order $p \geq 3$, any vertex of degree at least 2 is a cut vertex. There are at least two vertices of degree 1.
- The vertices of degree 1 in a tree are called **end vertices** or **leaves**. The remaining vertices are called **internal vertices**.
- Every edge in a tree is a bridge.

SPANNING TREES

- Every connected graph G contains a spanning subgraph that is a tree, called a **spanning tree**.
- A spanning tree that preserves the distance from v to each of the remaining vertices in G is said to be **distance preserving from v** or **v-distance preserving**.

DISTANCE PRESERVING TREES

Theorem 3.1.4: For every vertex v of a connected graph G, there exists a v-distance preserving tree.

MANY TREES EMBEDDED AS SUBGRAPHS

Theorem 3.1.5: Let G be a graph with $\delta(G) \geq m$ and let T be any tree of order $m + 1$. Then T is a subgraph of G.