

Section 1.4
Algorithms

ALGORITHM

An **algorithm** is a step-by-step procedure for solving a problem. In graph theory, the problems will be posed in terms of several **parameters** (or variables). An **instance** of a problem is obtained when we specify values for the parameters.

TIME COMPLEXITY

We will be most concerned with **time complexity**; that is the relative time it will take to perform an algorithm. We will try to measure the number of computational steps involved in the algorithm rather than the actual time for the algorithm to run.

ON THE ORDER OF

We say that the time complexity, $T(n)$, of an algorithm is **on the order of** $f(n)$ if there exist constants k and m such that $|T(n)| \leq k|f(n)|$ for all $n \geq m$. We notate this by $T(n) = O(f(n))$, which is read "big oh of $f(n)$ ".

NOTE: n is a parameter for the input size of the problem being considered.

POLYNOMIAL TIME COMPLEXITY

An algorithm has **polynomial time complexity** if its computational time complexity $T(n) = O(p(n))$ for some polynomial p in the input size n .

COMPARISON OF DIFFERENT TIME COMPLEXITIES

The table compares the times for various bounding functions based on given input sizes. This table assumes that any operations requires 0.000001 seconds.

	problem size			
time	10	30	50	100
n	0.00001 sec	0.00003 sec	0.00005 sec	0.0001 sec
n^2	0.0001 sec	0.0009 sec	0.0025 sec	0.01 sec
n^5	0.1 sec	24.3 sec	5.2 min	2.7 hrs
2^n	0.001 sec	17.9 min	25.7 yrs	2^{48} cent
3^n	0.059 sec	6.5 yrs	2×10^8 cent	3^{70} cent

INTRACTABLE PROBLEM

A problem for which no polynomial time algorithm can exist is called an [intractable problem](#).

DECISION PROBLEM

A [decision problem](#) is a problem that can be answered “yes” or “no”. For example, given two graphs G and H , does G contain a subgraph isomorphic to H .

CLASS NP PROBLEMS

- A problem is said to be [in the class \$NP\$](#) if it can be solved by a nondeterministic polynomial algorithm.
- For our purposes, a nondeterministic algorithm consists of two parts.
 - A guessing stage where a possible solution is “guessed”
 - A checking stage where the solution is checked to see if it provides a yes or no to the decision problem

CLASS NP PROBLEMS (CONCLUDED)

- All problems with polynomial solutions P are a subset of NP .
- It is not known if $P = NP$, although many believe this is not the case.
- If a problem is not in P but in NP it sits between polynomial time problems and intractable problems.

NP-COMPLETE PROBLEMS

A problem X is in the class of NP-complete problems if a solution for X provides a solution for all other problems Y in NP . By this we mean that there is a polynomial time algorithm to “transform” X into Y . Many graph theoretic problems are known to be in the class of NP -complete problems.
