Section 1.3

Alternate Representations for Graphs

ADJACENCY MATRIX

Let $G = (V, E)$ be a (p, q)-graph. Consider the $p \times p$ matrix $A = [a_{i\,j}]$, where each row and each column of A corresponds to a distinct vertex of V. Let $a_{i\,j} = 1$ if vertex v_i is adjacent to vertex v_j in G and $a_{i\,j} = 0$ otherwise. Note that $a_{i\,i} = 0$ for each $i = 1, 2, \ldots, p$. This matrix is called the **adjacency matrix** of G.

ADJACENCY MATRICES AND WALKS

Theorem 1.3.1: If A is the adjacency matrix of a graph G with vertices $v_1, v_2, \ldots, v_p$, then the (i, j)-entry of A^n is the number of $v_i - v_j$ walks of length n in G.

INCIDENCE MATRIX

For a (p, q) graph G, let the $p \times q$ matrix $M = [i_{x\,e}]$ be defined as follows: $i_{x\,e} = 1$ if vertex x is incident to edge e and $i_{x\,e} = 0$ otherwise. M is called the **incidence matrix** of G. The rows of M correspond to the vertices of G and the columns correspond to the edges of G.

ADJACENCY LIST

Let $G = (V, E)$. If we list each vertex in V along with those vertices that are adjacent to the listed vertex, we create an **adjacency list**. Nonadjacency is implied by omission from the list.