

## TEST I REVIEW MATH 4300

This is an outline of the main topics to be covered on Test I. It is not an exhaustive list. Thus, most of the questions in the exams usually come from the topics on this review but this cannot be guaranteed.

### Key Definitions and Main Concepts:

- Digraph, multi-digraph, pseudo-digraph, multi-pseudodigraph, in-degree & out-degree.
- Graph, multi-graph, pseudo-graph, multi-pseudograph, degree of a vertex, maximum degree ( $\Delta$ ) of a graph, minimum degree ( $\delta$ ) of a graph, degree sequence, sub-graph, induced subgraph, spanning subgraph, complete graphs, regular graphs, bipartite,  $k$ -partite graphs, isomorphic graphs.
- Adjacency matrix; geometric, set-theoretic, and matrix representations.
- Walk, trail, circuit, cycle, path, distances in weighted graphs; diameter, radius of a graph; connected graphs, the connected components, weakly & strongly connected digraphs, connectivity, cut vertex, edge connectivity, bridge.

### Main Algorithms:

- Graphical Sequence Algorithm & Graph Recovery Algorithm
- Breadth First Search Algorithm
- Depth First Search Algorithm
- Dijkstra's Distance Algorithm
- Ford's Distance Algorithm

### Main Results:

- The First Theorem of Graph Theory: Let  $G$  be a  $(p, q)$  graph and let  $V = \{v_1, v_2, \dots, v_p\}$ . Then  $\sum_{i=1}^p \deg v_i = 2q$ .
- Graphical Sequence Theorem: The decreasing sequence  $\langle d_1, d_2, d_3, \dots, d_n \rangle$  is graphical if and only if  $\langle d_2 - 1, d_3 - 1, \dots, d_{d_1+1} - 1, d_{d_1+2}, \dots, d_n \rangle$  is graphical.
- A connected graph with  $p$  vertices has at least  $p - 1$  edges.
- If  $G$  is a disconnected graph, then  $\bar{G}$  must be connected.
- If  $G$  has  $p$  vertices and  $\delta(G) > \frac{p-1}{2}$ , then  $G$  is connected.
- The number of walks of length  $n$  from  $v_i$  to  $v_j$  is  $A^n[i j]$  where  $A$  is the adjacency matrix for a graph  $G$ .
- Pigeon Hole Principle: If  $m$  pigeons are placed in  $k$  pigeon holes, then one hole will contain at least  $\left\lceil \frac{m}{k} \right\rceil$  pigeons.
- A degree sequence of a graph has at least two equal terms.
- Every  $x - y$  walk contains an  $x - y$  path.
- The edge  $e$  is a bridge if and only if  $e$  lies on no cycle.
- Whitney's Theorem: A graph  $G$  of order  $p \geq 3$  is 2-connected if and only if any two vertices of  $G$  lie on a common cycle.