Final Exam Review
MATH 4300

This is an outline of the main topics that have been covered since Test II. It is not an exhaustive list. ***The final exam will consists of topics from the reviews for Test I, Test II, and the list below***. Thus, most of the questions on the exams usually come from the topics on the reviews, but this cannot be guaranteed.

Key Definitions and Main Concepts:

- Planar graphs, planar embeddings, maximal planar graphs, K_5, $K_{3,3}$, segments, embeddability of a segment in a region, creating & merging out vertices of degree 2, shrinking edges, homeomorphisms.
- Matchings, maximum matching, perfect matching, (edge) cover, minimum (edge) cover, matchings in bipartite graphs, stable marriages.
- Vertex covers, independent vertices, legal colorings, chromatic number, the five color problem, the four color problem.

Main Algorithms:

- Pre-processing and the DMP Planarity Algorithm
- Using networks to find a maximum matching and a minimum cover in a bipartite graph
- Weighted Job Assignment Algorithm
- Stable Marriage Algorithm
- Largest First and Smallest Last Approximate Coloring Algorithms
- Brelaz Color-Degree Algorithm

Main Results:

- (a) If G is a connected planar graph, then $r = q + 2 - p$. (*Euler's formula*)
  (b) If G is a planar graph with k components, then $r = q + k + 1 - p$. (*Gen. Euler's formula*)
- Every region of a maximal planar graph is bounded by 3 edges.
- (a) In any maximal planar graph with more than 3 vertices, we have $q = 3p - 6$.
  (b) In any planar graph with more than 3 vertices, we have $q \leq 3p - 6$.
  (c) K_5 and $K_{3,3}$ are nonplanar.
- G is planar if an only if G has no subgraph which is homeomorphic to K_5 or $K_{3,3}$. (*Kuratowski's Theorem*)
- A matching M in a graph G is maximum if and only if there exists no M-augmenting path in G. (*Berge's Theorem*)
- (a) Let $G = (X \cup Y, E)$ be a bipartite graph. Then X can be matched to a subset of Y if and only if $|N(S)| \geq |S|$ for all subsets S of X. (*Hall's Theorem*)
  (b) If G is a k-regular bipartite graph with $k > 0$, then G has a perfect matching. (*Corollary to Hall's Theorem*)
- If $G = (X \cup Y, E)$ is a bipartite graph, then the maximum number of edges in a matching in G equals the minimum number vertices in a cover for $E(G)$; that is, $\beta_1(G) = \alpha(G)$. (*König-Egerváry Theorem*)
- If G is a critically n-chromatic graph, then $\delta(G) \geq n - 1$.
- In any graph G, $\chi(G) \leq \Delta(G) + 1$.
- If G is a connected graph that is neither an odd cycle nor a complete graph, then $\chi(G) \leq \Delta(G)$. (*Brooks' Theorem*)
- The Five Color Theorem and the Four Color Theorem.