# HOMEWORK

MATH 4300

Fall Semester 2019

<u>Chapter 1</u>: Due Friday, September 13
6, 10, 12, 13, 16, 19, 27, 28

<u>Chapter 2</u>: Due Friday, September 27
7, 10, 13, 15, 25, 26, 27, 28, 34

<u>Chapter 3</u>: Due Friday, October 11
1, 3, 6, 9, 11, 17, 18, 27, 28

<u>Chapter 4</u>: Due Friday, October 18
1, 5, 8, 9

- For #1, only use Ford and Fulkerson's algorithm and Dinic's algorithm.

<u>Chapter 5</u>: Due Monday, November 4
1, 4, 7, 11, 15, 16, 19

- For #4, only use Fleury's algorithm and Hierholzer's algorithm.
- For #7, just do the "if" direction; that is: Let $G$ be a nontrivial connected graph. If every edge of $G$ lies on an odd number of cycles, then $G$ is eulerian. HINT: Show every vertex has even degree. Count the number of cycles a vertex $v$ is contained in.
- For #19, the hypothesis should be "$G$ is hamiltonian connected and of order at least 4."

<u>Chapter 6</u>: Due Friday, November 15
1, 7, 9, 14, 15, 16 (first graph only)

<u>Chapter 7</u>: Due Friday, November 21
See attached assignment

<u>Chapter 8</u>: Monday, December 9
4, 5, 6, 7, 9, 11, 12, 16, 26 (c)

- For #12, not only state what the graphs are, but prove they work and are the only ones that work.
- For #26, be sure to state how you select the vertices.

**HOMEWORK PROBLEMS FOR CHAPTER 7**
MATH 4300
Due: Friday, November 21, 2019

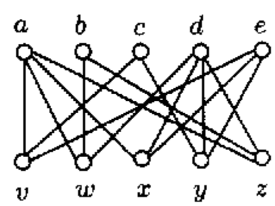
1. There are positions open in seven different divisions of a major company: advertising (a), business (b), computing (c), design (d), experimentation (e), finance (f), and guest relations (g). Six people are applying for some of these positions, namely:

| | |
|---|---|
| Alvin (A): a, c, f; | Beverly (B): a, b, c, d, e, g; |
| Connie (C): c, f; | Donald (D): b, c, d, e, f, g; |
| Enrique (E): a, c, f; | Frances (F): a, f. |

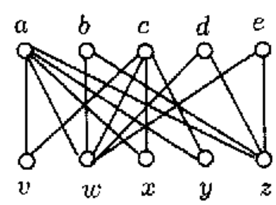
(a) Represent the situation by a bipartite graph.
(b) Is it possible to hire all six applicants for six different positions?

2. Two bipartite graphs $G_1$ and $G_2$ are shown below, each with partite sets $U = \{v, w, x, y, x\}$ and $W = \{a, b, c, d, e\}$. In each case, can $U$ be matched to $W$?

$G_1$ :




$G_2$ :




3. Four men and four women apply to a computer dating service. The computer evaluates the unsuitability of each man for each woman as a percentage (see the table below). Find the best possible dates for each woman for this Friday night.

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $W_1$ | 60 | 35 | 30 | 65 |
| $W_2$ | 30 | 10 | 55 | 30 |
| $W_3$ | 40 | 60 | 15 | 35 |
| $W_4$ | 25 | 15 | 40 | 40 |

**(continued on next page)**

4. Find men-optimal and women-optimal sets of stable marriages for the situation below.

| men | $w_1$ | $w_2$ | $w_3$ | $w_4$ |
|---|---|---|---|---|
| $m_1$ | 3 | 1 | 2 | 4 |
| $m_2$ | 4 | 1 | 3 | 2 |
| $m_3$ | 3 | 4 | 1 | 2 |
| $m_4$ | 1 | 4 | 2 | 3 |

| women | $w_1$ | $w_2$ | $w_3$ | $w_4$ |
|---|---|---|---|---|
| $m_1$ | 1 | 4 | 3 | 2 |
| $m_2$ | 4 | 3 | 2 | 1 |
| $m_3$ | 3 | 1 | 4 | 4 |
| $m_4$ | 2 | 2 | 1 | 3 |

5. Show that the $n$-cube $Q_n$ $(n \geq 2)$ has a perfect matching.

6. Prove that every tree has at most one perfect matching.

7. Let $G$ be a connected graph of order $2n$, where $n$ is a positive integer. Prove that if $G$ is $\{K_{1,3}\}$-free, then $G$ has a perfect matching. HINT: Induct on $n$ and consider a maximal path $P$ in $G$. Show $G$ minus the first two vertices of $P$ is connected.