

Section 8.4

Trigonometric Identities

WHAT IS AN IDENTITY?

Two functions f and g are **identically equal** if

$$f(x) = g(x)$$

for every value of x for which both functions are defined. Such an equation is referred to as an **identity**. An equation that is not an identity is called a **conditional equation**.

EXAMPLES

Simplify the following.

1. $(\cot \theta)(\sec \theta)$ by rewriting in terms of sine and cosine.
2. $\frac{\sin \theta}{1+\cos \theta}$ by multiplying by $\frac{1-\cos \theta}{1-\cos \theta}$
3. $\frac{1}{1-\cos \theta} + \frac{1}{1+\cos \theta}$ by getting a common denominator
4. Factor and simplify $\frac{\cos^2 \theta - 1}{\cos^2 \theta - \cos \theta}$

TIPS FOR ESTABLISHING A TRIGONOMETRIC IDENTITY

1. Work on the most complex side of the expression first.
2. Perform any indicated operations; *i.e.*, add fractions, squaring, etc.
3. Rewrite sums or differences of quotients as a single quotient. (You may need to get a common denominator.)
4. Factoring may also help.
5. Rewrite one side in terms of sine and cosine only.
6. Multiply the numerator and denominator of a fraction by the same factor.
7. **Work only on one side at a time.**

EXAMPLES

Establish the following trigonometric identities.

1. $\cot x + 1 = \csc x (\cos x + \sin x)$
2. $\sec x - \sin x = \frac{\csc x - \cos x}{\cot x}$
3. $\tan^2 x (1 + \cot^2 x) = \frac{1}{1 - \sin^2 x}$
4. $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$