

## Section 8.3

### Trigonometric Equations

### FINDING ALL SOLUTIONS TO TRIGONOMETRIC EQUATIONS

To find **all** solutions to a trigonometric equation, first find the solutions in the first revolution around the circle. Then add  $k2\pi$ , where  $k$  is any integer, to include all solutions coterminal with those already determined. This gives a general formula for all the solutions.

### EXAMPLES

Give a general formula for all solutions to the following trigonometric equations.

1.  $\tan \theta = \sqrt{3}$
2.  $2 \sin \theta + 2 = 0$
3.  $2 \cos^2 t - \cos t - 1 = 0$

### FIND SOLUTIONS IN THE INTERVAL $[0, 2\pi)$

It is very common to only find the solutions to trigonometric equations in the first revolution of the circle; that is, in the interval  $[0, 2\pi)$ .

**EXAMPLE:** Solve the following on the interval  $0 \leq \theta < 2\pi$ .

$$\cos^2 \theta - 3 \sin \theta + 3 \sin^2 \theta = 0$$

### COMMENT

Notice that in the last example we used a trigonometric identity to rewrite the equation so it could be solved more easily.

It is often necessary to use a trigonometric identity to aid in solving a trigonometric equation.

### SOLVING WHEN THE ARGUMENT IS OF THE FORM " $n\theta$ "

Some times in a trigonometric equation the trigonometric function has an argument of the form " $n\theta$ ," where  $n$  is a constant. To solve equations like this, solve for " $n\theta$ " and then solve for  $\theta$ .

To find the solutions in the interval  $0 \leq \theta < 2\pi$ , we substitute values in for  $k$  until the values of  $\theta$  become greater than  $2\pi$ .

## EXAMPLES

Solve the equation. Give a general formula for all the solutions. List the solutions in the interval  $0 \leq x < 2\pi$ .

1.  $\cos 2\theta = -\frac{1}{2}$
2.  $\tan 3x = -1$

## FINDING APPROXIMATE SOLUTION TO TRIGONOMETRIC EQUATIONS

Sometimes when solving trigonometric equations the trigonometric values are not from the common angles. To solve equations like these, we use inverse trigonometric functions to find one approximate solution. Then we use reference angles to find the other solutions.

## EXAMPLES

Find the approximate solutions to the following equations in the interval  $[0, 2\pi)$ .

1.  $\sin \theta - \frac{1}{3} = 0$
2.  $5 \cos^2 x - 4 \cos x - 1 = 0$