Sections 8.1 and 8.2

The Inverse Sine, Cosine, and Tangent Functions

Inverse Trigonometric Functions (Continued)

THE INVERSE SINE FUNCTION

Since $y = \sin x$ is not one-to-one, we must *restrict* its domain to find an inverse. We restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$

$$\text{where } -1 \leq x \leq 1 \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

DOMAIN AND RANGE OF INVERSE SINE

Consider the inverse sine function $y = \sin^{-1} x$.

(a) Its domain is $[-1, 1]$.

(b) Its range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

NOTE: Another notation for the inverse sine function is

$$y = \arcsin x$$

INVERSE SINE AND SINE

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x, \text{ where } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x, \text{ where } -1 \leq x \leq 1$$

THE INVERSE COSINE FUNCTION

Since $y = \cos x$ is not one-to-one, we must restrict its domain to find an inverse. We restrict the domain to $[0, \pi]$.

$$y = \cos^{-1} x \quad \text{means} \quad x = \cos y$$

$$\text{where } -1 \leq x \leq 1 \text{ and } 0 \leq y \leq \pi$$

DOMAIN AND RANGE OF INVERSE COSINE

Consider the inverse cosine function $y = \cos^{-1} x$.

(a) Its domain is $[-1, 1]$.

(b) Its range is $[0, \pi]$.

NOTE: Another notation for the inverse cosine function is

$$y = \arccos x$$

INVERSE COSINE AND COSINE

$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x$, where $0 \le x \le \pi$

$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x$, where $-1 \le x \le 1$

THE INVERSE TANGENT FUNCTION

Since $y = \tan x$ is not one-to-one, we must restrict its domain to find an inverse. We restrict the domain to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$y = \tan^{-1} x \quad \text{means} \quad x = \tan y$$

$$\text{where } -\infty < x < \infty \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}$$

DOMAIN AND RANGE OF INVERSE TANGENT

Consider the inverse tangent function $y = \tan^{-1} x$.

(a) Its domain is $(-\infty, \infty)$.

(b) Its range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

NOTE: Another notation for the inverse tangent function is

$$y = \arctan x$$

INVERSE TANGENT AND TANGENT

$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x$, where $-\infty \le x \le \infty$