

## Sections 7.6 and 7.8

### Graphs of Sine and Cosine Phase Shift

## VALUES FOR SINE

$x$	$y = \sin x$
0	0
$\pi/6$	0.5
$\pi/4$	0.7
$\pi/3$	0.9
$\pi/2$	1
$2\pi/3$	0.9
$3\pi/4$	0.7
$5\pi/6$	0.5
$\pi$	0

$x$	$y = \sin x$
$7\pi/6$	-0.5
$5\pi/4$	-0.7
$4\pi/3$	-0.9
$3\pi/2$	-1
$5\pi/3$	-0.9
$7\pi/4$	-0.7
$11\pi/6$	-0.5
$2\pi$	0

## PROPERTIES OF THE SINE FUNCTION

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from  $-1$  to  $1$ , inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period  $2\pi$ .
5. The  $x$ -intercepts are  $\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots$ ; the  $y$ -intercept is  $0$ .
6. The maximum value is  $1$  and occurs at

$$x = \dots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \dots; \text{ the minimum value is } -1 \text{ and occurs at } x = \dots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots.$$

## VALUES FOR COSINE

$x$	$y = \cos x$
0	1
$\pi/6$	0.9
$\pi/4$	0.7
$\pi/3$	0.5
$\pi/2$	0
$2\pi/3$	-0.5
$3\pi/4$	-0.7
$5\pi/6$	-0.9
$\pi$	-1

$x$	$y = \cos x$
$7\pi/6$	-0.9
$5\pi/4$	-0.7
$4\pi/3$	-0.5
$3\pi/2$	0
$5\pi/3$	0.5
$7\pi/4$	0.7
$11\pi/6$	0.9
$2\pi$	1

## PROPERTIES OF THE COSINE FUNCTION

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from  $-1$  to  $1$ , inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the  $y$ -axis indicates.
4. The cosine function is periodic, with period  $2\pi$ .
5. The  $x$ -intercepts are  $\dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$ ; the  $y$ -intercept is  $1$ .
6. The maximum value is  $1$  and occurs at  $x = \dots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \dots$ ; the minimum value is  $-1$  and occurs at  $x = \dots, -\pi, \pi, 3\pi, 5\pi, \dots$ .

## GRAPHS OF SINE AND COSINE

The period of sine and cosine is  $2\pi$ . The horizontal distance between the zeros and local extrema is  $\frac{\pi}{2}$ .

Observe that this is one-fourth of the period.

I generally classify zeros and local extrema as the "important points" of the graphs of sine and cosine.

### AMPLITUDE

The **amplitude** of the sine and cosine function is how far above or below the x-axis the graph goes. A change in the **amplitude** of sine or cosine results from a **vertical** stretch/compression (perhaps with a reflection).

The amplitude of

$$y = A \sin x \quad \text{and} \quad y = A \cos x$$

is  $|A|$ .

### PERIOD

A change in the **period** of a trigonometric function results from a **horizontal** stretch/compression.

The period,  $T$ , for

$$y = A \sin \omega x \quad \text{and} \quad y = A \cos \omega x$$

$$\text{is } T = \frac{2\pi}{\omega}.$$

### PHASE SHIFT

The **phase shift** of a trigonometric function results from a **horizontal** shift.

The phase shift of

$$y = A \sin(\omega x - \phi) \quad \text{and} \quad y = A \cos(\omega x - \phi)$$

$$\text{is } \frac{\phi}{\omega}.$$

### GRAPHING ONE PERIOD OF SINE AND COSINE

1. Find the amplitude, period, and phase shift.
2. Use the phase shift to determine the beginning of a period.
3. Add the phase shift and period to find the end of a period.
4. Divide the period by **4** to find the distance between "important points."
5. Plot the important points and sketch the graph.