Section 7.5

Unit Circle Approach; Properties of the Trigonometric Functions

THE UNIT CIRCLE

The circle that has its center at the origin and a radius of 1 is called the **unit circle**. Its equation is $x^2 + y^2 = 1$.

The unit circle has the property that the length of an arc of the circle is exactly the same as the radian measure of the angle that the arc subtends.

TRIGONOMETRIC FUNCTIONS OF REAL NUMBERS

Let t be *any* real number. To define the trigonometric functions of t, we measure t units around the unit circle starting from standard position. Let the point $P(x, y)$ be coordinates of the point on the unit circle corresponding to t. Then, the six trigonometric functions of the real number t are defined as follows:

$$\sin t = y \quad \cos t = x \quad \tan t = \frac{y}{x} \ (x \neq 0)$$

$$\csc t = \frac{1}{y} \quad \sec t = \frac{1}{x} \quad \cot t = \frac{x}{y} \ (y \neq 0)$$

$(y \neq 0) \quad (x \neq 0)$

TRIG. FUNCTIONS OF ANGLES AND THE UNIT CIRCLE

Since the unit circle has radius 1, the distance t around the unit circle is the same as the radian measure of the angle θ that subtends the arc. That is, $\theta = t$ radians. Thus, the six trigonometric functions of the angle θ are defined to be the six trigonometric of the real number t. That is,

$\sin \theta = \sin t \quad \cos \theta = \cos t \quad \tan \theta = \tan t$

$\csc \theta = \csc t \quad \sec \theta = \sec t \quad \cot \theta = \cot t$

DOMAINS AND RANGES OF THE TRIGONOMETRIC FUNCTIONS

For a summary of the domains and ranges see Table 6 on page 559 of the text.

A PROPERTY OF SINE AND COSINE

If we add (or subtract) integral multiples of 2π to θ, the sine and cosine values remain unchanged. That is, for all θ,

$$\sin(\theta + 2\pi k) = \sin \theta \quad \cos(\theta + 2\pi k) = \cos \theta$$

where k is any integer.

PERIODIC FUNCTIONS

A function f is called **periodic** if there is a number p such that, whenever θ is in the domain of f, so is $\theta + p$, and

$$f(\theta) = f(\theta + p)\,.$$

If there is a smallest such number p, this smallest value is called the **fundamental period** of f.

All six of the trigonometric functions are periodic.

PERIODS OF THE TRIGONOMETRIC FUNCTIONS

1. The period of $\cos\theta$, $\sin\theta$, $\sec\theta$, and $\csc\theta$ is 2π.
2. The period of $\tan\theta$ and $\cot\theta$ is π.

TRIGONOMETRIC FUNCTIONS AND EVEN AND ODD

The ***odd*** trigonometric functions are

$$f(\theta) = \sin\theta\,, f(\theta)\csc\theta$$
$$f(\theta) = \tan\theta,\ f(\theta) = \cot\theta$$

The ***even*** trigonometric functions are

$$f(\theta) = \cos\theta\,, f(\theta) = \sec\theta$$

Symbolically, this can be summarized as

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$