Section 7.4

Trigonometric Functions of General Angles

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

Let θ be any angle in standard position, and let (x, y) denote the coordinates of any point, except the origin $(0, 0)$, on the terminal side of θ. If $r = \sqrt{x^2 + y^2}$ denotes the distance from $(0, 0)$ to (x, y) then the six trigonometric functions of θ are defined as the ratios:

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \quad \sec\theta = \frac{r}{x} \quad \cot\theta = \frac{x}{y}$$

provided no denominator equals 0. If a denominator equals 0, that trigonometric function of the angle θ is not defined.

TRIGONOMETRIC FUNCTIONS OF QUADRANTAL ANGLES

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°; 0	0	1	0	not defined	1	not defined
90°; $\pi/2$	1	0	not defined	1	not defined	0
180°; π	0	−1	0	not defined	−1	not defined
270°; $3\pi/2$	−1	0	not defined	−1	not defined	0

COTERMINAL ANGLES

Two angles in standard position are said to be **coterminal** if they have the same terminal side

NOTE: Coterminal angles are ***NOT*** equal, they merely stop at the same place.

COTERMINAL ANGLES AND TRIGONOMETRIC FUNCTIONS

Because coterminal angles have the same terminal side, the values of the six trigonometric functions of coterminal angles are equal.

SIGNS OF THE TRIGONOMETRIC FUNCTIONS

Sign of	Terminal Side in Quadrant			
	I	II	III	IV
$\sin\theta$ $\csc\theta$	positive	positive	negative	negative
$\cos\theta$ $\sec\theta$	positive	negative	negative	positive
$\tan\theta$ $\cot\theta$	positive	negative	positive	negative

REFERENCE ANGLES

Let θ denote an angle that lies in a quadrant. The acute angle formed by the terminal side of θ and either the positive x-axis or the negative x-axis is called the **reference angle** for θ.

THE REFERENCE ANGLE THEOREM

Reference Angle Theorem: If θ is an angle, in standard position, that lies in a quadrant and α is its reference angle, then

$\sin\theta = \pm\sin\alpha \quad \cos\theta = \pm\cos\alpha \quad \tan\theta = \pm\tan\alpha$

$\csc\theta = \pm\csc\alpha \quad \sec\theta = \pm\sec\alpha \quad \cot\theta = \pm\cot\alpha$

where the + or − sign depends on the quadrant in which θ lies.

FINDING THE VALUES OF THE TRIGONOMETRIC FUNCTIONS OF ANY ANGLE

- If the angle θ is a quadrantal angle, draw the angle, pick a point on its terminal side, and apply the definition of the trigonometric functions.
- If the angle θ lies in a quadrant:
  1. Find the reference angle α of θ.
  2. Find the value of the trigonometric function at α.
  3. Adjust the sign (+ or −) of the value of the trigonometric functions based on the quadrant in which θ lies.