

Section 7.1

Angles and Their Measure

ANGLES

An **angle** is formed by rotating a ray about its endpoint. The original ray is the **initial side** of the angle and the second ray is the **terminal side** of the angle. The common endpoint is called the **vertex** of the angle.

POSITIVE AND NEGATIVE ANGLES

- An angle formed by a **counterclockwise** rotation is considered to be a **positive angle**.
- An angle formed by a **clockwise** rotation is considered to be a **negative angle**.

ANGLES AND THE xy -PLANE

An angle drawn on the xy -plane is in **standard position** when its initial side is on the positive x -axis.

MEASURE OF ANGLES

Angles are most commonly measured in **degrees**. One degree (1°) is $\frac{1}{360}$ of a complete revolution.

Degrees can be subdivided into **minutes** (notated by $'$) and **seconds** (notated by $''$) as follows:

$$1^\circ = 60' \quad 1' = 60''$$

CLASSIFICATION OF ANGLES

- 90° angles are **right angles**.
- 180° angles are **straight angles**.
- Angles that have a measure between 0° and 90° are **acute angles**.
- Angles that have a measure between 90° and 180° are **obtuse angles**.

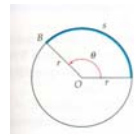
ANGLE CLASSIFICATION AND THE xy -PLANE

- If the terminal side of an angle in standard position lies on a coordinate axis, then the angle is called a **quadrantal angle**.
- If the terminal side of an angle in standard position does not lie on a coordinate axis, then the angle is classified by the quadrant that contains the terminal side.

ANGLES AND CIRCLES

Consider the circle below with two radii. The angle formed is called the **central angle**. The part of the circle between A and B is an **arc** of the circle and is written \widehat{AB}

We say that \widehat{AB} **subtends** the angle θ .



THE RADIAN

Another common measure of angles is the **radian**.

One **radian** is the measure of the central angle subtended by an arc of length r on a circle of radius r . Another way to think about this is measuring the distance of the radius along the edge of a circle.

RADIAN MEASURE

Given an arc of length s on a circle of radius r , the measure of the central angle subtended by the arc is $\theta = \frac{s}{r}$ radians.

Radian measure is simply how many radii you must go around the circle's edge to get the angle.

Radians are "unitless."

CONVERSIONS

To convert from radians to degrees, multiply by $\frac{180^\circ}{\pi}$.

To convert from degrees to radians, multiply by $\frac{\pi}{180^\circ}$.

ARC LENGTH

Let r be the length of the radius of a circle and θ be the nonnegative radian measure of a central angle of the circle. Then the length of the arc s that subtends the central angle is

$$s = r\theta .$$

EXAMPLES

1. Find the length of the arc that subtends an angle of 135° on a circle of radius 15 cm.
2. Suppose a bucket on a well is being raised and the “drum” has a radius of 5 inches. How many inches will the bucket raise if the drum is rotated through an angle of 75° ?
3. For the well in Example 2, suppose it takes 100 revolutions to raise the bucket from the bottom of the well. Approximately, how deep, in feet and inches, is the well?