Section 6.8

Exponential Growth and Decay;
Newton's Law of Cooling;
Logistic Models

UNIHIBITED POPULATION GROWTH

A model that gives the population N after a time t has passed (in the early stages of growth) is

$$N(t) = N_0 \cdot e^{kt}, \qquad k > 0$$

Where $N_0 = N(0)$ is the initial population and k is a positive constant that represents the growth rate.

EXAMPLES

1. In 2010 the population of the world was 6.9 billion. In 2012 the world population was 7.1 billion. Find an equation for the population growth and use it to predict the world population in 2018.
2. A colony of bacteria is growing exponentially. If the colony has a population of 2500 at noon and 2600 at 4:00 pm, how long will it take the population to double?

UNIHIBITED RADIOACTIVE DECAY

The amount A of a radioactive material present at time t is given by

$$A(t) = A_0 \cdot e^{kt}, \qquad k < 0$$

Where A_0 is the original amount of radioactive material and k is a negative number that represents the rate of decay.

EXAMPLES

3. A radioactive substance has a half-life of 810 years. If there were 10 grams initially, how much would be left after 300 years?
4. All living things contain carbon-12, which is stable, and carbon-14, which is radioactive. While a plant or animal is alive, the ratio of these two isotopes of carbon remains unchanged since the carbon-14 is constantly renewed; after death, no more carbon-14 is absorbed. The half-life of carbon-14 is 5730 years. If a human bone found in an archeological dig is found to only have 39% of the carbon-14 of living tissue, how long ago did the person die?

NEWTON'S LAW OF COOLING

Newton's Law of Cooling states that the temperature of a heated object decreases exponentially over time toward the temperature of the surrounding medium.

Newton's Law of Cooling: The temperature u of a heated object at a given time t can be modeled by the following function:

$$u(t) = T + (u_0 - T) \cdot e^{kt}, \qquad k < 0$$

where T is the temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

EXAMPLE

5. A pot of coffee with a temperature of 100°C is set down in a room with a temperature of 20°C. The coffee cools to 60°C after 1 hour.
   (a) Find the values for T, u_0, and k.
   (b) Find temperature of the coffee after half an hour.
   (c) How long did it take the coffee to reach 50°C?