Section 6.1
Section 6.2

Composite Functions

One-to-One Functions; Inverse Functions

THE COMPOSITE FUNCTION

Given two function f and g, the **composite function**, denote by $f \circ g$ (read "f composed with g"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

ONE-TO-ONE FUNCTIONS

A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. Put another way, a function is one-to-one if no y in the range is the image of more than one x in the domain.

TESTING FOR A ONE-TO-ONE FUNCTION

Horizontal Line Test: A function f is one-to-one if and only if every horizontal line intersects the graph of f in at most one point.

A THEOREM ON ONE-TO-ONE FUNCTIONS

- A function that is increasing on an interval I is a one-to-one function on I.
- A function that is decreasing on an interval I is a one-to-one function on I.

DEFINITION OF AN INVERSE FUNCTION

Suppose that f is a one-to-one function. Then, corresponding to each x in the domain of f, there is exactly one y in the range (because f is a function); and corresponding to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of f**. The symbol f^{-1} is used to denote the inverse function of f.

NOTE: $f^{-1}(x)$ does ***NOT*** mean $\frac{1}{f(x)}$

PROPERTIES OF INVERSE FUNCTIONS

Domain of f = Range of f^{-1}; that is, $D_f = R_{f^{-1}}$

Range of f = Domain of f^{-1}; that is, $R_f = D_{f^{-1}}$

AN EQUIVALENT DEFINITION OF INVERSE FUNCTIONS

In the language of function notation, two functions f and f^{-1} are **inverses** of each other if and only if

$(f^{-1} \circ f)(x) = f^{-1}\big(f(x)\big) = x$ where x is in the domain of f

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$ where x is in the domain of f^{-1}

GRAPHING AN INVERSE FUNCTION

Theorem: The graph of a one-to-one functions f and the graph of its inverse function f^{-1} are symmetric with respect to the line $y = x$.

Practical Application: Given the graph of a one-to-one function, the graph of its inverse is obtained by switching x- and y-coordinates.

FINDING A FORMULA FOR AN INVERSE FUNCTION

To find a formula for the inverse given an equation for a one-to-one function:

1. Replace $f(x)$ with y.
2. Interchange x and y.
3. Solve the resulting equation for y.
4. Replace y with $f^{-1}(x)$.