Section 3.4

Library of Functions;
Piecewise-Defined Functions

THE SQUARE ROOT FUNCTION

Properties of $f(x) = \sqrt{x}$

1. The domain and range are both the set of nonnegative real numbers; that is, $[0, \infty)$.
2. The x-intercept of the graph is $(0, 0)$. The y-intercept of the graph is also $(0, 0)$.
3. The function is neither even nor odd.
4. It is increasing on the interval $(0, \infty)$.
5. It has a local minimum of $y = 0$ at $x = 0$.

THE CUBE ROOT FUNCTION

Properties of $f(x) = \sqrt[3]{x}$

1. The domain and range are the set of all real numbers; that is, $(-\infty, \infty)$.
2. The x-intercept is $(0, 0)$. The y-intercept is also $(0, 0)$.
3. The function is odd.
4. It is increasing on the interval $(-\infty, \infty)$.
5. It does not have a local minimum or a local maximum.

THE ABSOLUTE VALUE FUNCTION

Properties of $f(x) = |x|$

1. The domain is the set of all real numbers; that is, $(-\infty, \infty)$. The range is all nonnegative real numbers; that is, $[0, \infty)$.
2. The x-intercept of the graph is $(0, 0)$. The y-intercept of the graph is also $(0, 0)$.
3. The function is even.
4. It is decreasing on the interval $(-\infty, 0)$. It is increasing on the interval $(0, \infty)$.
5. It has a local minimum of $y = 0$ at $x = 0$.

LIBRARY OF FUNCTIONS

1. **Constant Function:** $f(x) = b$
   Domain: $(-\infty, \infty)$
   Its graph is a horizontal line.
   Its y-intercept is $(0, b)$.
   It is constant on the entire domain.

2. **Identity Function:** $f(x) = x$
   Domain: $(-\infty, \infty)$
   Its graph is a line.
   Its y-intercept is $(0, 0)$.
   It is increasing on the entire domain.

LIBRARY (CONTINUED)

3. **Linear Function:** $f(x) = mx + b$
   Domain: $(-\infty, \infty)$
   Its graph is a line.
   Its y-intercept is $(0, b)$.
   Increasing if $m > 0$. Decreasing if $m < 0$.
   It is the constant function if $m = 0$.
   It is the identity function if $m = 1$ and $b = 0$.

4. **Square Function:** $f(x) = x^2$
   Domain: $(-\infty, \infty)$
   Its graph is a parabola.
   Its y-intercept is $(0, 0)$.
   It is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

LIBRARY (CONTINUED)

5. **Cube Function:** $f(x) = x^3$
   Domain: $(-\infty, \infty)$
   Its y-intercept is $(0, 0)$.
   It is increasing on $(-\infty, \infty)$.

6. **Square Root Function:** $f(x) = \sqrt{x}$
   Domain: $[0, \infty)$
   Its graph is a half parabola.
   Its y-intercept is $(0, 0)$.
   It is increasing on the interval $(0, \infty)$.

LIBRARY (CONTINUED)

7. **Cube Root Function:** $f(x) = \sqrt[3]{x}$
   Domain: $(-\infty, \infty)$
   Its y-intercept is $(0, 0)$.
   It is increasing.

8. **Reciprocal Function:** $f(x) = \frac{1}{x}$
   Domain: $(-\infty, 0) \cup (0, \infty)$
   It has no intercepts.
   It is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$.

LIBRARY (CONTINUED)

9. **Absolute Value Function:** $f(x) = |x|$
   Domain: $(-\infty, \infty)$
   Its y-intercept is $(0, 0)$.
   It is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

10. **Greatest Integer Function:** $f(x) = \text{int}(x)$
    Domain: $(-\infty, \infty)$
    It is an example of a *step function*.
    Its y-intercept is $(0, 0)$.
    It is constant on the intervals $[n, n + 1)$ where n is any integer.

PIECEWISE-DEFINED FUNCTIONS

A **piecewise-defined function** is a function that is defined by more than one equation. An example is

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ |x| & \text{if } -1 < x \leq 1 \\ x - 2 & \text{if } x > 1 \end{cases}$$