Section 3.3

Properties of Functions

INTERCEPTS OF FUNCTIONS

- The ***y*-intercept** of the graph of a function $y = f(x)$ is the value of the function when $x = 0$, provided 0 is in the domain. That is, the y-intercept is $f(0)$. Graphically, the y-intercept is where the graph crosses (or touches) the y-axis.
- The ***x*-intercepts** of the graph of a function $y = f(x)$ are the solutions of the equation $f(x) = 0$. There do *not* have to be any x-intercepts. Graphically, the x-intercepts are the places where the graph crosses (or touches) the x-axis. Sometimes x-intercepts are called ***zeros*** of the function.

EVEN AND ODD FUNCTIONS

- A function is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = f(x)$$

- A function is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = -f(x)$$

SYMMETRY AND EVEN AND ODD FUNCTIONS

Theorem:

- A function is **even** if and only if its graph is **symmetric with respect to the *y*-axis**.
- A function is **odd** if and only if its graph is **symmetric with respect to the origin**.

INCREASING AND DECREASING FUNCTIONS

- A function is **increasing** on an open interval I when as x-values get larger the y-values get larger in the interval. That is, the graph goes from lower left to upper right in the interval I.
- A function is **decreasing** on an open interval I when as x-values get larger the y-values get smaller on the interval. That is, the graph goes from upper left to lower right in the interval I.
- A function is **constant** on an interval I when as the x-values get larger the y-values are the same on the interval. That is, the graph is a horizontal line on the interval.

MAXIMA AND MINIMA

- A y-value is a **local maximum** if it is the largest y-value in its neighborhood. A y-value is an **absolute maximum** if it is the largest y-value of the entire function/graph.
- A y-value is a **local minimum** if it is the smallest y-value in its neighborhood. A y-value is an **absolute minimum** if it is the smallest y-value of the entire function/graph.

THE EXTREME VALUE THEOREM

Theorem: If f is a continuous function whose domain is a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

AVERAGE RATE OF CHANGE

If c is in the domain of a function $y = f(x)$, the **average rate of change of *f*** from c to x is defined as

$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}, \qquad x \neq c$$

This formula is a variation of the **difference quotient** studied in Section 3.1.

THE SECANT LINE

The line containing two points on a graph is called a **secant line**.

Theorem: The average rate of change of a function equals the slope of the secant line containing the two points on its graph. That is, the slope of the line through the points $(x, f(x))$ and $(c, f(c))$ is

$$m_{\text{sec}} = \frac{f(x) - f(c)}{x - c}$$