Section 10.2

Polar Equations and Graphs

HORIZONTAL AND VERTICAL LINES

1. The graph of $r \sin\theta = a$ is a horizontal line a units above the pole if a is positive and $|a|$ units below the pole if a is negative.
2. The graph of $r \cos\theta = a$ is a vertical line a units to the right of the pole if a is positive and $|a|$ units to the left of the pole if a is negative.

POLAR EQUATIONS OF CIRCLES

- The equation $r = a$ is a circle of radius $|a|$ centered at the pole.
- The equation $r = a\cos\theta$ is a circle of radius $\left|\frac{a}{2}\right|$, passing through the pole, and with center on $\theta = 0$ or $\theta = \pi$.
- The equation $r = a\sin\theta$ is a circle of radius $\left|\frac{a}{2}\right|$, passing through the pole, and with center on $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$.

ROSE CURVES

The equations

$$r = b\sin(a\theta)$$
$$r = b\cos(a\theta)$$

both have graphs that are called **rose curves**.

- The rose curve has **$2a$** leaves (petals) if a is an even number.
- The rose curve has **a** leaves (petals) if a is an odd number.
- The leaves (petals) have length **b**.

LIMAÇONS

The graphs of the equations

$$r = a \pm b\sin\theta$$
$$r = a \pm b\cos\theta$$

are called **limaçons**.

- If $\left|\frac{a}{b}\right| < 1$, then the limaçon has an inner loop. For example: $r = 3 - 4\cos\theta$.
- If $\left|\frac{a}{b}\right| = 1$, then the limaçon is a "heart-shaped" graph called a **cardiod**. For example: $r = 3 - 3\sin\theta$.

LIMAÇONS (CONTINUED)

- If $1 < \left|\frac{a}{b}\right| < 2$, then the limaçon is dimpled. For example: $r = 3 + 2\cos\theta$.
- If $\left|\frac{a}{b}\right| \geq 2$, then the limaçon is convex. For example: $r = 3 + \sin\theta$.