STUDY GUIDE FOR TEST IV

MATH 1113

1. Change each logarithmic expression to an equivalent expression involving an exponent.
   (a) $\log_5 u = 13$ (b) $\log_a 4 = 3$

2. Change each exponential expression into an equivalent expression involving a logarithm.
   (a) $5^2 = z$ (b) $a^5 = m$

3. Use translations (shifts) of $f(x) = 5^x$ and $g(x) = \log_5 x$ to graph the following functions. State the domain and range. Identify any asymptotes.
   (a) $y = 5^x + 1$ (b) $y = 5^{x-1}$
   (c) $y = 5^{x+1} - 2$ (d) $y = \log_5 x - 5$
   (e) $y = \log_5(x - 2)$ (f) $y = \log_5(x + 1) + 2$

4. Solve the following equations.
   (a) $9^x = 27$ Solve ***without*** using a calculator.
   (b) $2^{3x-1} = \frac{1}{8}$ Solve ***without*** using a calculator.
   (c) $8^x = 24$ (d) $10^{2x+1} = 280$
   (e) $3e^{2x-1} = 10$ (f) $\log_3 x^2 = 6$
   (g) $\log_7(x - 3) = 1 - \log_7 x$ (h) $\ln 3 + \ln(x - 4) = \ln(x + 2)$
   (i) $\log 10^{4x-3} = 7$

5. Write each expression as a single logarithm.
   (a) $\log x - 2\log 6 + 5\log y$ (b) $3\ln t - 3(\ln u - 2\ln v)$
   (c) $3\log_2 x + \log_2(x^3 + 2) - \frac{1}{2}\log_2(3x^2 + 2)$

6. Rewrite each expression to have no logarithms except $\log_7 x$.
   (a) $\log_7 \frac{1}{\sqrt[3]{x}}$ (b) $\log_7 \frac{x}{49}$
   (c) $\log_7(x^3 + 9x^2) - \log_7(x + 9)$

7. Write each expression as a sum and/or difference of logarithms. Express powers as factors.

   (a) $\log_6\left(\frac{y^2}{xz^5}\right)$ (b) $\log_2\left(\frac{\sqrt{z}y^2}{x^2}\right)$

   (c) $\ln\left(\frac{\sqrt{x-2}\sqrt[3]{x+1}}{x^2+3}\right)$

8. Using algebra, find the domain of the following functions. Check your answer on your graphing calculator.

   (a) $g(x) = \ln(4 + x)$ (b) $f(x) = \log_3(x^2 - 9)$

   (c) $f(x) = \log(x^2)$ (d) $g(x) = \log_5(-x^2 + 3x + 10)$

9. Antonia Lopez currently has $67,894 in a savings account that pays 5.22% annual interest compounded monthly.

   (a) If she neither deposits nor withdraws from the account, how much money will she have in seven years?

   (b) How much more would she have if the interest were compounded continuously?

10. Kevin wishes to deposit $3,000 for a period of two years for a special purpose. After visiting several banks and investigating his options, he has selected three plans:

    A. 6.5% interest per year compounded daily
    B. 6.55% interest per year compounded monthly
    C. 6.6% interest per year compounded quarterly

    (a) Which plan will earn Kevin the most money?

    (b) How much will he lose if he chooses the worst plan?

11. Jim invests $22,000 of his retirement savings in CD that pays 6.85% annual interest compounded monthly. When will the CD be worth $35,000? Hint: Use logarithms.

12. In a certain bacteria culture the rate of growth is proportional to the number of bacteria present. If the initial population is 5000 bacteria and after 20 minutes the population is 10,000, how long will it take the population to reach 25,000?

13. A town grew exponentially from 10,000 in 2000 to 14,000 in 2010. Assuming the same type of growth continues, what will be the population in 2030?

14. The half-life of radium is 1600 years. How much of a 200-milligram sample will be left after 100 years?

15. Strontium 90 ($_{90}$Sr) has a half-life of 28 years. How of a 55-milligram sample with remain after 5 years?

16. Suppose a canned soft drink is placed in a refrigerator that maintains a constant temperature of 34°F. Suppose it takes the soft drink 5 minutes to cool from 75°F to 65°F.
    (a) Find an equation for the cooling of the soft drink.
    (b) What will be the temperature (to the nearest degree) of the soft drink after 30 minutes?
    (c) When (to the nearest minute) will the temperature of the soft drink be 36°F?

17. A thermometer is taken from a room at 71°F to the outdoors where the temperature is 14°F. Suppose the reading on the thermometer drops to 45°F after 1 minute.
    (a) Find an equation for the cooling of the thermometer.
    (b) What will be the temperature reading (to the nearest degree) of the thermometer after 3 minutes?
    (c) When will the temperature reading of the thermometer be 20°F? Round your answer to the nearest tenth of a minute.

18. Graph the following conic sections. Give the following information for each conic section.
    Parabola: vertex, axis of symmetry, focus, and directrix.
    Ellipse: center, vertices, covertices, major axis, minor axis, foci, and eccentricity.
    Circle: center and radius.
    Hyperbola: center, vertices, transverse axis, conjugate axis, foci, and eccentricity.

    (a) $(x+1)^2 + (y-2)^2 = 9$
    (b) $(y+2)^2 = -(x-1)$
    (c) $\frac{(y-1)^2}{4} - \frac{(x-2)^2}{16} = 1$
    (d) $\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$

19. Put the following equations of conic sections in standard graphing form by completing the square. Then graph the conic sections. Give the following information for each conic section.
    Parabola: vertex, axis of symmetry, focus, and directrix.
    Ellipse: center, vertices, covertices, major axis, minor axis, foci, and eccentricity.
    Circle: center and radius.
    Hyperbola: center, vertices, transverse axis, conjugate axis, foci, and eccentricity.

    (a) $9x^2 - 4y^2 - 90x - 64y - 67 = 0$
    (b) $4x^2 + 9y^2 + 32x - 54y - 179 = 0$
    (c) $x^2 + y^2 + 2x + 6y - 12 = 0$
    (d) $-2x^2 - 8x + y - 9 = 0$

20. Find an equation in standard form of the indicated conic section satisfying the given criteria.
    (a) A parabola with vertex (1, 3) and focus at (4, 3).
    (b) A hyperbola with vertices (−2, 2) and (4, 2) and eccentricity 5/3.
    (c) A circle with center (2, −3) and passing through the point (4, −4).
    (d) An ellipse with vertices at (3, 2) and (3, −4) and foci $\left(3, \sqrt{5} - 1\right)$ and $\left(3, -\sqrt{5} - 1\right)$.

ANSWERS

1. (a) $5^{13} = u$ (b) $a^3 = 4$

2. (a) $\log_5 z = 2$ (b) $\log_a m = 5$

3. (a) D: $(-\infty, \infty)$; R: $(1, \infty)$; h.a.: $y = 1$
   (b) D: $(-\infty, \infty)$; R: $(0, \infty)$; h.a.: $y = 0$
   (c) D: $(-\infty, \infty)$; R: $(-2, \infty)$; h.a.: $y = -2$
   (d) D: $(0, \infty)$; R: $(-\infty, \infty)$; v.a.: $x = 0$
   (e) D: $(2, \infty)$; R: $(-\infty, \infty)$; v.a.: $x = 2$
   (f) D: $(-1, \infty)$; R: $(-\infty, \infty)$; v.a.: $x = -1$

4. (a) $x = \frac{3}{2}$ (b) $x = -\frac{2}{3}$
   (c) $x = \frac{\ln 24}{\ln 8} = \frac{\log 24}{\log 8}$ (d) $x = \frac{\log 280 - 1}{2}$
   (e) $x = \frac{\ln \frac{10}{3} + 1}{2}$ (f) $x = \pm 27$
   (g) $x = \frac{3 + \sqrt{37}}{2}$ (h) $x = 7$
   (i) $x = \frac{5}{2}$

5. (a) $\log\left(\frac{xy^5}{36}\right)$ (b) $\ln\left(\frac{t^3 v^6}{u^3}\right)$
   (c) $\log_2\left[\frac{x^3(x^3+2)}{\sqrt{3x^2+2}}\right]$

6. (a) $-\frac{1}{3}\log_7 x$ (b) $-2+\log_7 x$
   (c) $2\log_7 x$

7. (a) $2\log_6 y - \log_6 x - 5\log_6 z$ (b) $\frac{1}{2}\log_2 z + 2\log_2 y - 2\log_2 x$
   (c) $\frac{1}{2}\ln(x-2) + \frac{1}{3}\ln(x+1) - \ln(x^2+3)$

8. (a) $(-4, \infty)$ (b) $(-\infty, -3) \cup (3, \infty)$
   (c) $(\infty, 0) \cup (0, \infty)$ (d) $(-2, 5)$

9. (a) \$97,763.88 (b) \$77.50

10. (a) Plan C (b) \$3.19

11. approximately 6.8 years

12. approximately 46.4 minutes

13. 27,401 people

14. approximately 192 mg

15. approximately 48.6 mg

16. (a) $u(t) = 34 + 41e^{-0.056t}$ (b) 42°F
    (c) 54 minutes

17. (a) $u(t) = 14 + 57e^{-0.609t}$ (b) 23°F
    (c) 3.7 minutes

18. (a) circle; center: $(-1, 2)$; radius: 3
    (b) parabola opening left; vertex: $(1, -2)$; axis of symmetry: $y = -2$; focus: $\left(\frac{3}{4}, -2\right)$; directrix: $x = \frac{5}{4}$
    (c) hyperbola; center: $(2, 1)$; vertices: $(2, 3), (2, -1)$; transverse axis: $x = 2$; foci: $\left(2, 1+2\sqrt{5}\right), \left(2, 1-2\sqrt{5}\right)$; $e = \sqrt{5}$

(d) ellipse; center: (−2, 1), vertices: (−2, 4), (−2, −2), covertices: (0, 1), (−4, 1), major axis: $x = -2$, minor axis: $y = 1$, foci: $\left(-2, 1+\sqrt{5}\right)$, $\left(-2, 1-\sqrt{5}\right)$; $e = \frac{\sqrt{5}}{3}$

19. (a) hyperbola: $\frac{(x-5)^2}{4} - \frac{(y+8)^2}{9} = 1$
center: (5, −8); vertices: (7, −8), (3, −8); transverse axis: $y = -8$;
foci: $\left(5+\sqrt{13}, -8\right), \left(5-\sqrt{13}, -8\right)$; $e = \frac{\sqrt{13}}{2}$

(b) ellipse: $\frac{(x+4)^2}{81} + \frac{(y-3)^2}{36} = 1$
center: (−4, 3); vertices: (−13, 3), (5, 3); covertices: (−4, 9), (−4, −3);
major axis: $y = 3$, minor axis: $x = -4$; foci: $\left(-4+3\sqrt{5}, 3\right), \left(-4-3\sqrt{5}, 3\right)$;
$e = \frac{\sqrt{5}}{3}$

(c) circle: $(x+1)^2 + (y+3)^2 = 22$
center: (−1, −3); radius: $\sqrt{22}$

(d) parabola opening up: $(x+2)^2 = \frac{1}{2}(y-1)$
vertex: (−2, 1); axis of symmetry: $x = -2$; focus: $\left(-2, \frac{9}{8}\right)$; directrix: $y = \frac{7}{8}$

20. (a) $(y-3)^2 = 12(x-1)$ (b) $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{16} = 1$

(c) $(x-2)^2 + (y+3)^2 = 5$ (d) $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$