STUDY GUIDE FOR TEST III

MATH 1113

Comment on TEST III: Only the following identities will be provided:

Sum and difference formulas

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

Double-angle formulas

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

Half-angle formulas

$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$

1. Use trigonometric identities to write each expression in terms of a single trigonometric function.

(a) $1+\frac{\sin^2 x}{\cos^2 x}$

(b) $\frac{\tan x+1}{\cot x+1}$

(c) $\frac{\cos^2 x+\sin^2 x}{\csc x}$

(d) $(\sin^2 x)(\tan^2 x+1)$

(e) $1+\frac{1}{\tan^2 x}$

(f) $\frac{\cos^2 x}{1-\sin^2 x}-1$

2. Verify the following trigonometric identities.

(a) $\frac{\sec x}{\tan x} = \csc x$

(b) $\sec^2 x \cot x = \sec x \csc x$

(c) $\sin x \cos^2 x + \sin^3 x = \sin x$

(d) $\cos x \cot x + \sin x = \csc x$

(e) $\frac{\cos x}{1+\sin x} = \frac{1-\sin x}{\cos x}$

(f) $\frac{\sin^2 x}{1+\cos x} = 1 - \cos x$

3. Use the sum or difference identities to find the exact value of the following expressions.

(a) $\sin(60° + 45°)$

(b) $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

(c) $\cos(135° + 30°)$

(d) $\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$

4. Use the sum, difference, or double-angle formulas to write the following expressions as a single trigonometric function.

(a) $\sin 3x \cos 4x + \cos 3x \sin 4x$

(b) $\cos 6x \cos x + \sin 6x \sin x$

(c) $2 \sin 4\theta \cos 4\theta$

(d) $2 \cos^2 2\theta - 1$

5. (a) Find the exact value of $\sin 2\theta$ given that $\cos\theta = -\frac{12}{13}$ and $\frac{\pi}{2} < \theta < \pi$.

(b) Find the exact value of $\cos 2\theta$ given that $\sin\theta = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$.

6. Find the exact value of each of the following expressions. Do NOT use a calculator.

(a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

(c) $\tan^{-1} 1$

(d) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$

(e) $\tan\left(\tan^{-1}\frac{1}{\pi}\right)$

(f) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

(g) $\cos^{-1}\left(\sin\frac{2\pi}{3}\right)$

(h) $\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$

(i) $\sin\left[\cos^{-1}\left(\frac{5}{11}\right)\right]$

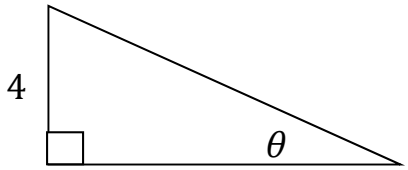
7. Solve the following equations involving inverse trigonometric functions.
   (a) $2\sin^{-1}(x-1) = \frac{\pi}{3}$
   (b) $\cos^{-1} x = \sin^{-1}\frac{5}{13}$
   (c) $\sin^{-1} x + \cos^{-1}\frac{4}{5} = \frac{\pi}{2}$

8. Solve each equation on the interval $0 \le x < 2\pi$. Give the ***exact*** solutions, not decimal approximations.
   (a) $2\sin x - 1 = 0$
   (b) $2\cos^2 x - \cos x - 1 = 0$
   (c) $2\sin^2 x = 1$
   (d) $\sin^2 x - \cos^2 x = \sin x$

9. Solve each equation. Give a general formula for ***all*** solutions. Give the ***exact*** solutions, not decimal approximations.
   (a) $\cos 3x + \frac{1}{2} = 0$
   (b) $(\sin x)(1 - \tan x) = 0$

10. Solve the following problems. Do ***NOT*** use a calculator.
    (a) A 50-foot ladder is leaning against a vertical stone wall. If the top of the ladder makes an angle of 30° with the wall, how many feet is the bottom of the ladder from the base of the wall?
    (b) Video screens are measured by the length of the diagonal of the screen. Find the length of the base of a 36-inch video screen if the diagonal makes an angle of 45° with the base of the screen.
    (c) If the angle of elevation of the sun is 60°, how long is the shadow cast by a tree that is 120 feet tall?

11. Solve the following problems. You may use a calculator.
    (a) At a point 449 feet distant from a radio tower, the angle of elevation of the top of the tower is 69.2°. Approximate the height of the tower to the nearest tenth of a foot.
    (b) A skateboard ramp is to have a grade of 10%, which means $\tan\theta = 0.10$. One end is to be raised 4 feet above the level ground. See the diagram below. How long a board should be used to make the ramp?



Skateboard Ramp

    (c) An airplane traveling at 150 mph is descending at an angle of depression of 8°. How many feet will the plane descend in 20 minutes?

12. Solve the following triangles. You may use a calculator.
    (a) $a = 12, b = 15, c = 20$
    (b) $b = 102, c = 150, A = 82°$
    (c) $a = 8, c = 10, A = 105°$
    (d) $c = 25, A = 55°, B = 80°$
    (e) $b = 9.0, c = 14, B = 32°$

13. Find the area of each triangle. Round each area accurate to two significant figures. You may use a calculator.
    (a) $a = 60$ m, $b = 44$ m, $C = 44°$
    (b) $b = 50$ in, $c = 75$ in, $C = 15°$
    (c) $a = 24$ cm, $b = 30$ cm, $c = 36$ cm

ANSWERS

1. (a) $\sec^2 x$
   (b) $\tan x$
   (c) $\sin x$
   (d) $\tan^2 x$
   (e) $\csc^2 x$
   (f) 0

2. The answer is in working the problems.

3. (a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (b) $\frac{\sqrt{2}+\sqrt{6}}{4}$
   (c) $\frac{-\sqrt{6}-\sqrt{2}}{4} = -\frac{\sqrt{6}+\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}+\sqrt{2}}{4}$

4. (a) $\sin 7x$ (b) $\cos 5x$
   (c) $\sin 8\theta$ (d) $\cos 4\theta$

5. (a) $-\frac{120}{169}$ (b) $-\frac{7}{25}$

6. (a) $-\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$
   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{4}$
   (e) $\frac{1}{\pi}$ (f) $\frac{\pi}{3}$
   (g) $\frac{\pi}{6}$ (h) $-\frac{3}{4}$
   (i) $\frac{4\sqrt{6}}{11}$

7. (a) $\frac{3}{2}$ (b) $\frac{12}{13}$
   (c) $\frac{4}{5}$

8. (a) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ (b) $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$
   (c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ (d) $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

9. (a) $\left\{\frac{2\pi}{9} + \frac{k2\pi}{3}, \frac{4\pi}{9} + \frac{k2\pi}{3}, \text{where } k \text{ is an integer}\right\}$
   (b) $\left\{k\pi, \frac{\pi}{4} + k\pi, \text{where } k \text{ is an integer}\right\}$

10. (a) 25 feet (b) $18\sqrt{2}$ inches
    (c) $40\sqrt{3}$ feet

11. (a) 1182.0 feet (b) 40.2 feet
    (c) approximately 37,000 feet

12. (a) $A \approx 37°, B \approx 48°, C \approx 95°$ (b) $a \approx 169, B \approx 37°, C \approx 61°$
    (c) No triangle formed (d) $a \approx 29, b \approx 35, C \approx 45°$
    (e) $a_1 \approx 17, A_1 \approx 92°, C_1 \approx 56°$ or
    $a_2 \approx 6.9, A_2 \approx 24°, C_2 \approx 124°$

13. (a) ≈ 920 m^2 (b) ≈ 790 in^2
    (c) ≈ 360 cm^2