STUDY GUIDE FOR TEST II

MATH 1113

Comments on TEST II:

(i) ***No calculators will be allowed on this test.*** As a result, all answers on the test should be in ***exact*** (non-decimal) form.

(ii) Be sure to know the domains and ranges of all the trigonometric functions.

(iii) Be sure to know which of the trigonometric functions are even and which ones are odd.

(iv) Be sure to know in which quadrants the trigonometric functions are positive and negative.

1. Find the complement of each of the following angles.

   (a) $33°$ (b) $68°$

   (c) $\frac{3\pi}{8}$ (d) $\frac{2\pi}{11}$

2. Convert the following angles from degrees to radians or radians to degrees. Leave your answers in ***exact*** form.

   (a) $-45°$ (b) $15°$

   (c) $315°$ (d) $630°$

   (e) $\frac{\pi}{4}$ (f) $-\frac{2\pi}{3}$

   (g) $\frac{\pi}{9}$ (h) $\frac{11\pi}{18}$

   (i) $\frac{6\pi}{5}$

3. Find the radian measure of the central angle of the circle with radius 40 centimeters and an arc length of 12 centimeters.

4. Sketch the following directed angles on the unit circle. For quadrantal angles, use the unit circle. For the non-quadrantal angles, use reference angles (and "the chart") to find the ***exact value*** of the six trigonometric functions of θ without using your calculator.

   (a) $\theta = \frac{13\pi}{6}$ (b) $\theta = -225°$

   (c) $\theta = \frac{4\pi}{3}$ (d) $\theta = 540°$

   (e) $\theta = \frac{7\pi}{2}$

5. Draw a right triangle containing the acute angle θ, and find the ***exact value*** of the remaining five trigonometric functions of θ without using your calculator.

   (a) $\sec\theta = \frac{3}{2}$ (b) $\cot\theta = \frac{24}{7}$

6. Find the values of the five remaining trigonometric functions of θ given:

   (a) $\tan\theta = \frac{5}{4};\ \pi < \theta < \frac{3\pi}{2}$ (b) $\sin\theta = -\frac{2}{5};\ \frac{3\pi}{2} < \theta < 2\pi$

7. Find the ***exact value*** of each expression. Do not use a calculator.

   (a) $\sin 45° \cos 45°$ (b) $\sec 30° \cot 45°$

   (c) $2\sin\frac{\pi}{3} - 3\tan\frac{\pi}{6}$ (d) $\tan\frac{\pi}{3} + \cos\frac{\pi}{3}$

   (e) $\tan\pi - \cos 0$ (f) $\sec\pi - \csc\frac{\pi}{2}$

8. Use trigonometric identities to find the ***exact value*** of each of the following. Do not use a calculator.

   (a) $\sin^2 20° + \frac{1}{\sec^2 20°}$ (b) $\frac{1}{\cos^2 40°} - \frac{1}{\cot^2 40°}$

   (c) $\tan 10° \cot 10°$ (d) $\frac{\sin 50°}{\cos 40°}$

   (e) $\tan(-20°)\cot 20°$ (f) $\sin 400° \sec(-50°)$

9. Find the amplitude, period, and phase shift of the function, sketch one period of the function, and label x-intercepts and local minima and local maxima with their coordinates.

   (a) $y = \frac{1}{4}\sin(4x - \pi)$ (b) $y = -\cos\left(\frac{x}{2}\right)$

   (c) $y = 3\sin\left(2x + \frac{\pi}{2}\right)$

10. Find the period of the following trigonometric functions. Sketch one period of the graph. Label any asymptotes with their equations. Also, label any x-intercepts and local minima and maximum with their coordinates.

    (a) $y = 2\tan 2x$ (b) $y = \frac{1}{4}\cot\frac{x}{6}$

    (c) $y = -3\sec 6x$ (d) $y = \csc\frac{x}{4}$

ANSWERS

1. (a) $57°$ (b) $22°$
   (c) $\frac{\pi}{8}$ (d) $\frac{7\pi}{22}$

2. (a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{12}$
   (c) $\frac{7\pi}{4}$ (d) $\frac{7\pi}{2}$
   (e) $45°$ (f) $-120°$
   (g) $20°$ (h) $110°$
   (i) $216°$

3. $\theta = \frac{3}{10}$

4. (a) $\sin\theta = \frac{1}{2};\ \csc\theta = 2$
   $\cos\theta = \frac{\sqrt{3}}{2};\ \sec\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
   $\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3};\ \cot\theta = \sqrt{3}$

   (b) $\sin\theta = \frac{\sqrt{2}}{2};\ \csc\theta = \sqrt{2}$
   $\cos\theta = -\frac{\sqrt{2}}{2};\ \sec\theta = -\sqrt{2}$
   $\tan\theta = -1;\ \cot\theta = -1$

   (c) $\sin\theta = -\frac{\sqrt{3}}{2};\ \csc\theta = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$
   $\cos\theta = -\frac{1}{2};\ \sec\theta = -2$
   $\tan\theta = \sqrt{3};\ \cot\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

   (d) $\sin\theta = 0;\ \csc\theta$ undefined
   $\cos\theta = -1;\ \sec\theta = -1$
   $\tan\theta = 0;\ \cot\theta$ undefined

   (e) $\sin\theta = -1;\ \csc\theta = -1$
   $\cos\theta = 0;\ \sec\theta$ undefined
   $\tan\theta$ undefined; $\cot\theta = 0$

5. (a) $\sin\theta = \frac{\sqrt{5}}{3};\ \csc\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$
   $\cos\theta = \frac{2}{3};\ \sec\theta = \frac{3}{2}$
   $\tan\theta = \frac{\sqrt{5}}{2};\ \cot\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

   (b) $\sin\theta = \frac{7}{25};\ \csc\theta = \frac{25}{7}$
   $\cos\theta = \frac{24}{25};\ \sec\theta = \frac{25}{24}$
   $\tan\theta = \frac{7}{24};\ \cot\theta = \frac{24}{7}$

6. (a) $\sin\theta = -\frac{5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$; $\csc\theta = -\frac{\sqrt{41}}{5}$
$\cos\theta = -\frac{4}{\sqrt{41}} = -\frac{4\sqrt{41}}{41}$; $\sec\theta = -\frac{\sqrt{41}}{4}$
$\tan\theta = \frac{5}{4}$; $\cot\theta = \frac{4}{5}$

(b) $\sin\theta = -\frac{2}{5}$; $\csc\theta = -\frac{5}{2}$
$\cos\theta = \frac{\sqrt{21}}{5}$; $\sec\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$
$\tan\theta = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$; $\cot\theta = -\frac{\sqrt{21}}{2}$

7. (a) $\frac{1}{2}$
(b) $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
(c) 0
(d) $\sqrt{3} + \frac{1}{2}$
(e) -1
(f) -2

8. (a) 1
(b) 1
(c) 1
(d) 1
(e) -1
(f) 1

9. (a) amplitude: $\frac{1}{4}$; period: $\frac{\pi}{2}$; phase shift: $\frac{\pi}{4}$
(b) amplitude: 1; period: 4π; phase shift: none (0)
(c) amplitude: 3; period: π; phase shift: $-\frac{\pi}{4}$

10. (a) period: $\frac{\pi}{2}$
(b) period: 6π
(c) period: $\frac{\pi}{3}$
(d) period: 8π