STUDY GUIDE FOR TEST I

MATH 1113

1. Determine which of the following are functions are functions of x. Give a reason to support your answer. If it is a function, state the domain and range. (Assume all tables are complete.)

(a)

x	y
3	1
7	11
3	−3
−1	5

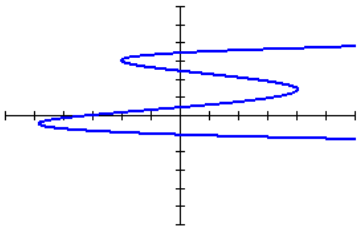
(b)

x	y
4	2
2	7
7	−2
−2	4

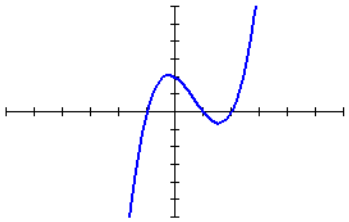
(c) $y = \frac{1}{3}x^3 - 8$

(d) $x^2 - y^2 = 1$

(e)



(f)



2. Determine the domain of the following functions.

(a) $y = x^2 - 4x + 3$

(b) $y = \sqrt{3 - x}$

(c) $y = \frac{x+2}{x^2-6x-27}$

(d) $y = \frac{1}{\sqrt{x^2-5x+4}}$

3. Use <u>algebra</u> to determine whether the following functions are even, odd, or neither. Check your answer by graphing and looking for symmetry.

(a) $f(x) = -x^4 + x^2 - 99$

(b) $f(x) = -x^2 + x^3$

(c) $f(x) = (-x^3 + 4x)(x^2 + 1)$

(d) $f(x) = |x|$

(e) $f(x) = 3x^4 - 4x + 5$

(f) $f(x) = x^3 + 3x$

(g) $f(x) = \sqrt{1 - x^2}$

(h) $f(x) = |x + 3|$

4. Given $f(x) = \begin{cases} |x| & \text{if} \quad x \leq 2 \\ 2 & \text{if} \quad x > 2 \end{cases}$.

(a) Find $f(-2)$, $f(0)$, $f(2)$, and $f(3)$.

(b) Sketch the graph of f.

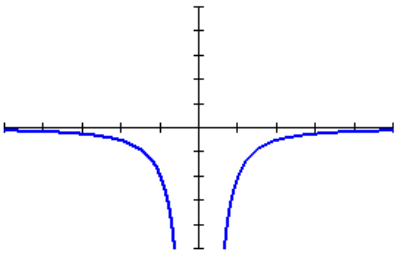
5. Given $f(x) = \begin{cases} -x - 1 & \text{if} \quad x \leq -2 \\ x^2 - 3 & \text{if} \quad -2 < x < 0 \\ -3 & \text{if} \quad x \geq 0 \end{cases}$.

(a) Find $f(-2)$, $f(-1)$, $f(0)$, and $f(2)$.

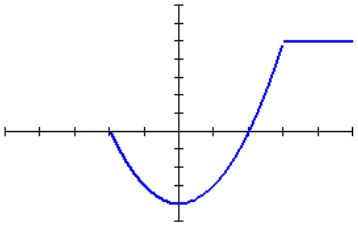
(b) Sketch the graph of f.

6. Determine the intervals (x-interval) in the domain on which the following functions are increasing, decreasing, and/or constant.

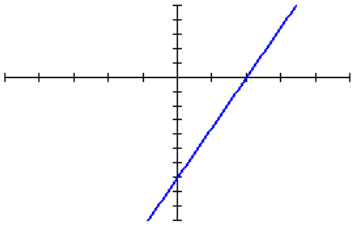
(a)



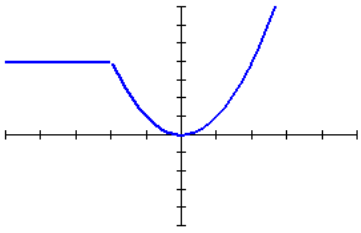
(b)



(c)



(d)



7. Determine whether the graphs of the following equations are symmetric about the x-axis, the y-axis, the origin, or none of the above.

(a) $x = y^2 - 4$ (b) $y = x^2 + 4$

(c) $y = x^4 - 3x^2 - 5$ (d) $y^2 - (x+1)^2 = 1$

(e) $y = x^3 - x$ (f) $\frac{x^2}{25} + \frac{y^2}{81} = 1$

8. Let $f(x) = x^2 + 2x - 3$ and $g(x) = \frac{1}{x+3}$. Compute the following, simplify if necessary, and state the domain.

(a) $f + g$ (b) $f - g$

(c) fg (d) $\frac{f}{g}$

9. Compute $f \circ g$ and $g \circ f$ for each of the pairs of functions below. Also, state the domain of $f \circ g$ and $g \circ f$.

(a) $f(x) = 1 - 3x; \quad g(x) = \frac{1}{3x^2}$

(b) $f(x) = x^3 - 36; \; g(x) = \sqrt{x} - 10$

(c) $f(x) = \frac{1}{x-10}; \quad g(x) = \sqrt{x}$

10. Verify by computing $f \circ g$ and $g \circ f$ that $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{2x}{1-x}$ are inverses.

11. Sketch the graph of each of the following functions and determine whether or not each function has an inverse. If it has an inverse, sketch the graph of the inverse.

(a) $f(x) = x^2 - 1$ (b) $f(x) = \sqrt{x-5}$

(c) $f(x) = \frac{x^2}{x-1}$

12. Find the inverse of the following functions. Sketch the graph of the inverse by using the graph of $f(x)$.

(a) $f(x) = \frac{x-1}{x+1}$

(b) $f(x) = \sqrt[3]{2x+1}$

13. Compute the difference quotient for the following functions. Use the difference quotient formula $\frac{f(x+h)-f(x)}{h}, h \neq 0$.

(a) $f(x) = 2x + 1$

(b) $f(x) = 3x^2 - 2x$

(c) $f(x) = x^2 + x + 1$

14. If $f(x) = x^2 - x - 2$, find

(a) $f(3)$

(b) $f(-5)$

(c) $f(-x)$

(d) $f(t)$

(e) $f(2t)$

(f) $f(t-3)$

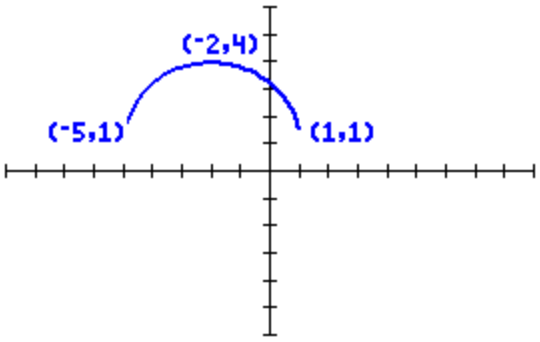
(g) $f(2t+1)$

(h) If $x = 8$, what is $f(x)$?

(i) If $f(x) = 4$, what is x?

(j) If $f(x) = 10$, what is x?

15. The graph of $F(x)$ is shown below. Sketch the graph of the following.



(a) $y = F(x-4) - 3$

(b) $y = \frac{1}{3}F(x)$

(c) $y = F\left(\frac{1}{3}x\right)$

(d) $y = -F(x)$

(e) $y = F(-x)$

(f) $y = -2F(x)$

16. Find the x-values where the following functions are discontinuous, identify each as a vertical asymptote or missing point, find the horizontal asymptote (if any). Find any oblique asymptote. Sketch a graph of the function. Plot and label the x- and y-intercepts. Use dashed lines to represent asymptotes.

(a) $R(x) = \frac{1}{x^2-1}$ (b) $R(x) = \frac{3x^2}{x^2+1}$

(c) $R(x) = \frac{x^2-x-6}{2x^2+x-1}$ (d) $R(x) = \frac{3x}{3x^2-x-2}$

(e) $R(x) = \frac{x^3-27}{x-3}$ **Hint:** The numerator factors. See Section R.5, Example 3, page 44.

(f) $R(x) = \frac{2x^2-x-8}{x+2}$ (g) $R(x) = \frac{3x^2-7x-6}{2x^2-7x+3}$

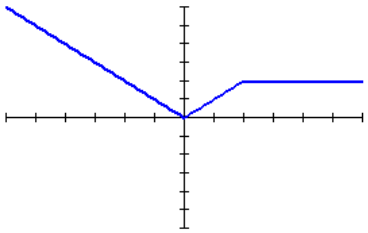
ANSWERS

1. (a) no
   (b) yes; D: {4, 2, 7, −2}; R: {2, 7, −2, 4}
   (c) yes; D: $(-\infty, \infty)$; R: $(-\infty, \infty)$
   (d) no
   (e) no
   (f) yes; D: $(-\infty, \infty)$; R: $(-\infty, \infty)$

2. (a) D: $(-\infty, \infty)$
   (b) D: $(-\infty, 3]$
   (c) D: $(-\infty, -3) \cup (-3, 9) \cup (9, \infty)$
   (d) D: $(-\infty, 1) \cup (4, \infty)$

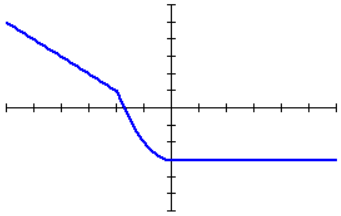
3. (a) even (symmetric wrt the y-axis)
   (b) neither
   (c) odd (symmetric wrt the origin)
   (d) even (symmetric wrt the y-axis)
   (e) neither
   (f) odd (symmetric wrt the origin)
   (g) even (symmetric wrt the y-axis)
   (h) neither

4. (a) 2, 0, 2, 2
   (b)



5. (a) 1, –2, –3, –3

(b)



6. (a) increasing: $(0, \infty)$
decreasing: $(-\infty, 0)$

(b) increasing: $(0, 3)$
decreasing: $(-2, 0)$
constant: $(3, \infty)$

(c) increasing: $(-\infty, \infty)$

(d) increasing: $(0, \infty)$
decreasing: $(-2, 0)$
constant: $(-\infty, -2)$

7. (a) *x*-axis
(b) *y*-axis
(c) *y*-axis
(d) *x*-axis
(e) origin
(f) *x*-axis, *y*-axis, and origin

8. (a) $x^2 + 2x - 3 + \frac{1}{x+3}$; D: $(-\infty, -3) \cup (-3, \infty)$
(b) $x^2 + 2x - 3 - \frac{1}{x+3}$; D: $(-\infty, -3) \cup (-3, \infty)$
(c) $\frac{x^2+2x-3}{x+3} = x - 1$; D: $(-\infty, -3) \cup (-3, \infty)$
(d) $(x^2 + 2x - 3)(x + 3) = x^3 + 5x^2 + 3x - 9$; D: $(-\infty, -3) \cup (-3, \infty)$

9. (a) $(f \circ g)(x) = 1 - \frac{1}{x^2}$; D: $(-\infty, 0) \cup (0, \infty)$
$(f \circ g)(x) = \frac{1}{3(1-3x)^2}$; D: $\left(-\infty, \frac{1}{3}\right) \cup (1\backslash 3, \infty)$
(b) $(f \circ g)(x) = \left(\sqrt{x} - 10\right)^3 - 36$; D: $[0, \infty)$
$(g \circ f)(x) = \sqrt{x^3 - 36} - 10$; D: $\left[\sqrt[3]{36}, \infty\right)$
(c) $(f \circ g)(x) = \frac{1}{\sqrt{x}-10}$; D: $[0,100) \cup (100, \infty)$
$(g \circ f)(x) = \frac{1}{\sqrt{x-10}}$; D: $(10, \infty)$

10. Note: The answer is in working the problem.

11. (a) no
(b) yes
(c) no

12. (a) $f^{-1}(x) = \frac{x+1}{1-x}$ (b) $f^{-1}(x) = \frac{x^3-1}{2}$

13. (a) 2 (c) $6x + 3h - 2$
(c) $2x + h + 1$

14. (a) 4 (b) 28
(c) $x^2 + x - 2$ (d) $t^2 - t - 2$
(e) $4t^2 - 2t - 2$ (f) $t^2 - 7t + 10$
(g) $4t^2 + 2t - 2$ (h) 54
(i) $x = -2,\ x = 3$ (j) $x = -3, x = 4$

15 (a) shifts right by 4 and down by 3 (b) vertical compression by factor of $\frac{1}{3}$
(c) horizontal stretch by a factor of 3 (d) reflection across *x*-axis
(e) reflection across the *y*-axis (f) reflection and stretch by factor of 2 across *x*-axis.

16. (a) v.a.: $x = 1,\ x = -1$; h.a.: $y = 0$; *x*-int: none; *y*-int: $(0, -1)$
(b) no points of discontinuity; h.a.: $y = 3$; *x*-int and *y*-int: $(0, 0)$
(c) v.a.: $y = \frac{1}{2},\ x = -1$; h.a.: $y = \frac{1}{2}$; *x*-int: $(-2, 0), (3, 0)$; *y*-int: $(0, 6)$
(d) v.a.: $x = -\frac{2}{3},\ x = 1$; h.a.: $y = 0$; *x*-int and *y*-int: $(0,0)$
(e) missing point at $x = 3$; *x*-int: none; *y*-int: $(0,9)$
(f) v.a.: $x = -2$; o.a.: $y = 2x - 5$; *x*-int: $\left(\frac{1+\sqrt{65}}{4}, 0\right), \left(\frac{1-\sqrt{65}}{4}, 0\right)$; *y*-int: $(0, -4)$
(g) v.a.: $x = \frac{1}{2}$; missing point at $x = 3$; h.a.: $y = \frac{3}{2}$; *x*-int: $\left(-\frac{2}{3}, 0\right)$; *y*-int: $(0, -2)$.