SECTION 5.3 – Examples

Analyze and graph the following functions.

① $R(x) = \dfrac{x^2+x-12}{x^2-4}$

STEP 1: $R(x) = \dfrac{(x+4)(x-3)}{(x+2)(x-2)}$

Set denom $= 0$

$(x+2)(x-2) = 0$

$x+2=0 \qquad x-2=0$

$x=-2 \qquad x=2$

Domain is all reals except -2 and 2

D: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

STEP 2: R is already in lowest terms

STEP 3: x-int: (set y=0)

$\dfrac{x^2+x-12}{x^2-4}=0$ (multiply both sides by denominator)

$x^2+x-12 = 0(x^2-4)$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x+4=0 \quad x-3=0$

$x=-4 \quad x=3$

x-intercepts: $(-4,0)$ and $(3,0)$

y-int: (Set x=0) $\quad R(0) = \dfrac{0^2+0-12}{0-4} = -\dfrac{12}{-4} = 3$

y-intercept: $(0,3)$

STEP 4: vertical asymptotes: $x=2$, $x=-2$ (where denominator was zero in step 2).

STEP 5: Since deg of numerator and degree of denominator are equal (both are 2), there is a horizontal asymptote at $y = \frac{1}{1} = 1$.

Now determine if the graph crosses the h.a. by setting equation equal to the value of the horizontal asymptote and solving the equation

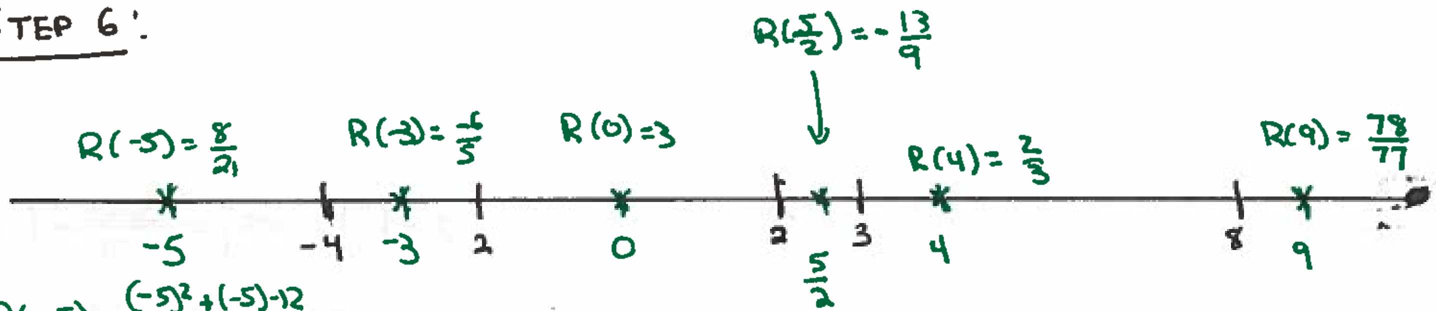
$$\frac{x^2+x-12}{x^2-4} = 1 \Rightarrow x^2+x-12 = 1(x^2-4) \Rightarrow x^2+x-12 = x^2-4$$

$$x = x^2-4-x^2+12$$

$$x = 8$$

Thus, the point $(8,1)$ is on the graph.

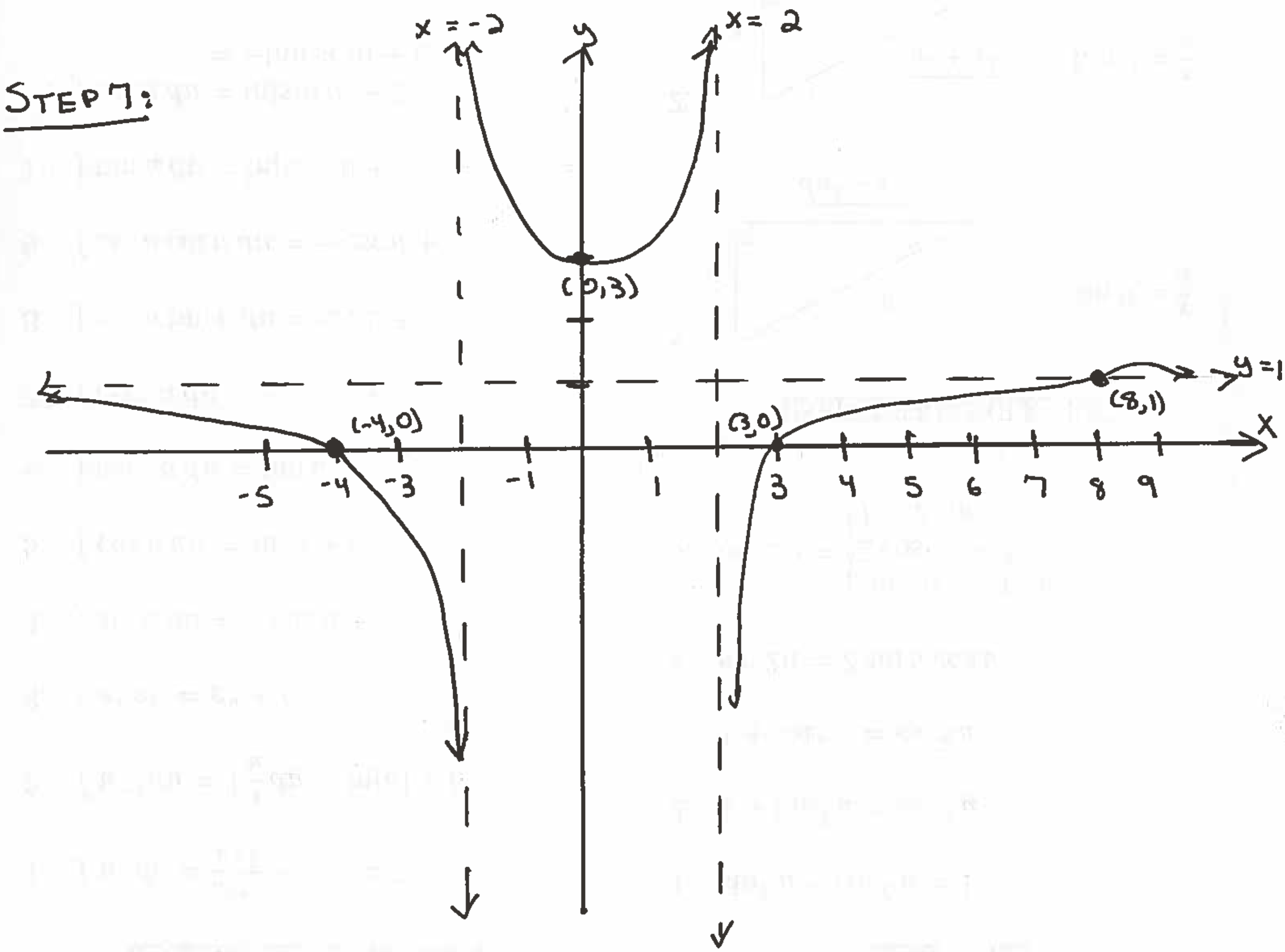
STEP 6:



$$R(-5) = \frac{(-5)^2 + (-5) - 12}{(-5)^2 - 4}$$

$$= \frac{8}{21}$$

STEP 7:



② $R(x) = \dfrac{3(x-1)(x+2)(x-3)}{x(x^2-4)} = \dfrac{3x^3-6x^2-15x+18}{x^3-4x}$

STEP 1: $R(x) = \dfrac{3(x-1)(x+2)(x-3)}{x(x-2)(x+2)}$

Set denom equal zero:

$x(x-2)(x+2)=0$

$x=0 \quad x-2=0 \quad x+2=0$

$x=0 \quad x=2 \quad x=-2$ → Domain is all real numbers except 0, 2, and -2.

$(-\infty,-2) \cup (-2,0) \cup (0,2) \cup (2,\infty)$

STEP 2: In lowest terms,

$R(x) = \dfrac{3(x-1)(x+2)(x-3)}{x(x-2)(x+2)} = \dfrac{3(x-1)(x-3)}{x(x-2)} \quad x \neq -2$

STEP 3: x-int: (Set y=0)

$\dfrac{3(x-1)(x-3)(x+2)}{x(x^2-4)} = 0$

$3(x-1)(x-3)(x+2)=0$

$x-1=0 \quad x-3=0 \quad x+2=0$

$x=1 \quad x=3 \quad x=-2$ → not an x-int. since $x=-2$ is not in the domain

X-intercepts: $(1,0), (3,0)$

y-int (set $x=0$) no y-intercepts since $x=0$ is not in the domain

STEP 4: To find vertical asymptotes, use the function in lowest terms from STEP 2. The vertical asymptotes will be where the denominator is 0.

$x(x-2)=0$

$x=0 \quad x-2=0$

$x=2$

V.a: $x=0 \quad x=2$

NOTE: Since $x=-2$ makes the denominator zero in the original equation but not the reduced equation, there is a missing point at $x=-2$

The "y-value" of the missing point is $\dfrac{3(-2-1)(-2-3)}{(-2)(-2-2)} = \dfrac{3(-3)(-5)}{-2(-4)} = \dfrac{45}{8}$

STEP 5: horizontal asymptote: $y = \frac{3}{1} = 3$

no oblique asymptote

To determine if the graph crosses the horizontal asymptote, set $R(x)$ equal to the value of the horizontal asymptote. In this case,

$$R(x) = 3$$

$$\frac{3(x-1)(x+2)(x-3)}{x(x^2-4)} = 3$$

$$\frac{3(x-1)(x+2)(x-3)}{x(x+2)(x-2)} = 3$$

$$\frac{3(x-1)(x-3)}{x(x-2)} = 3$$

$$\frac{3(x-1)(x-3)}{3} = \frac{3x(x-2)}{3}$$

$$(x-1)(x-3) = x(x-2)$$

$$x^2 - 4x + 3 = x^2 - 2x$$

$$x^2 - 4x + 3 - x^2 + 2x = 0$$

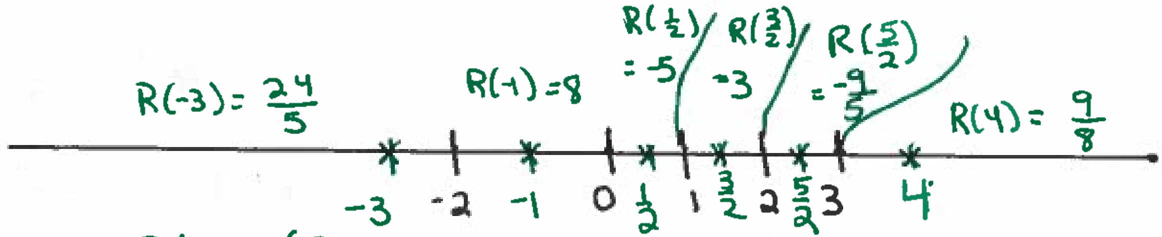
$$-2x + 3 = 0$$

$$\frac{-2x}{-2} = \frac{-3}{-2}$$

$$x = \frac{3}{2}$$

Thus, the point $(\frac{3}{2}, 3)$ is on the graph.

STEP 6:



$$R(-3) = \frac{3(-3-1)(-3+2)(-3-3)}{-3[(-3)^2-4]}$$

$$= \frac{24}{5}$$

STEP 7: