

MATH 1113 Formula Sheet

Properties of Logarithms:

$$\log_a x = y \text{ is equivalent to } a^y = x$$

$$\log_a a^r = r$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a M^r = r \log_a M$$

$$\log_a \frac{1}{N} = -\log_a N$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = P \cdot e^{rt}$

A = accumulated amount, P = principle, r = annual interest rate, t = number of years, n = number of annual compoundings

Uninhibited Population Growth:

$$N(t) = N_0 e^{kt}, \quad k > 0 \text{ where } N_0 = \text{initial population}$$

Radioactive Decay:

$$A(t) = A_0 e^{kt}, \quad k < 0 \text{ where } A_0 = \text{original amount}$$

Newton's Law of Cooling:

$u(t) = T + (u_0 - T)e^{kt}$, $k > 0$ where T = temperature of surrounding medium, u_0 = initial temperature of the object

Converting Angles:

$$\text{Degrees to Radians: } \frac{\pi}{180^\circ}$$

$$\text{Radians to Degrees: } \frac{180^\circ}{\pi}$$

Arc Length: $s = r\theta$, θ measured in radians

Trigonometric Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} \quad \sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta \quad \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \quad \sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \\ 1 - 2 \sin^2 \alpha \end{cases}$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\b^2 &= a^2 + c^2 - 2ac \cos B \\c^2 &= a^2 + b^2 - 2ab \cos C\end{aligned}$$

Area of a Triangle:

$$\begin{aligned}K &= \frac{1}{2}ab \sin C \\K &= \frac{1}{2}ac \sin B \\K &= \frac{1}{2}bc \sin A\end{aligned}$$

$$K = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

Polar to Rectangular:

$$x = r \cos \theta; \quad y = r \sin \theta; \quad \cos \theta = \frac{x}{\sqrt{x^2+y^2}}; \quad \sin \theta = \frac{y}{\sqrt{x^2+y^2}}; \quad r^2 = x^2 + y^2;$$

Rectangular to Polar:

$$r^2 = x^2 + y^2; \quad \tan \theta = \frac{y}{x}; \quad x = r \cos \theta; \quad y = r \sin \theta$$

Circle: $(x-h)^2 + (y-k)^2 = r^2$ where center = (h, k) and radius = r

Parabola: vertex = (h, k)

$$\begin{aligned}(x-h)^2 &= 4a(y-k) \text{ axis of symmetry is vertical} \\(y-k)^2 &= 4a(x-h) \text{ axis of symmetry is horizontal} \\ \text{Focus is } |a| \text{ units "inside" and directrix is } |a| \text{ units "outside"}\end{aligned}$$

Ellipse: center = (h, k)

$$\begin{aligned}\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} &= 1 \quad (a > b) \text{ major axis is horizontal} \\ \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} &= 1 \quad (a > b) \text{ major axis is vertical}\end{aligned}$$

vertices: a units from center in both directions

covertices: b units from center in both directions

foci: c units from center in both directions where $c^2 = a^2 - b^2$.

eccentricity: $e = \frac{c}{a}$

Hyperbola: center = (h, k)

$$\begin{aligned}\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} &= 1 \text{ transverse axis is horizontal} \\ \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} &= 1 \text{ transverse axis is vertical}\end{aligned}$$

vertices: a units from center in both directions

foci: c units from center in both directions where $c^2 = a^2 + b^2$.

eccentricity: $e = \frac{c}{a}$